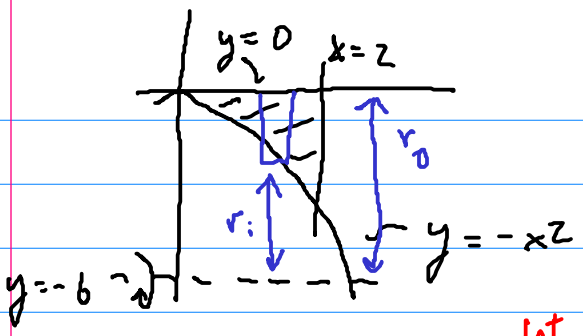
viSiiQg

1.




1pt 2 — 6 pts

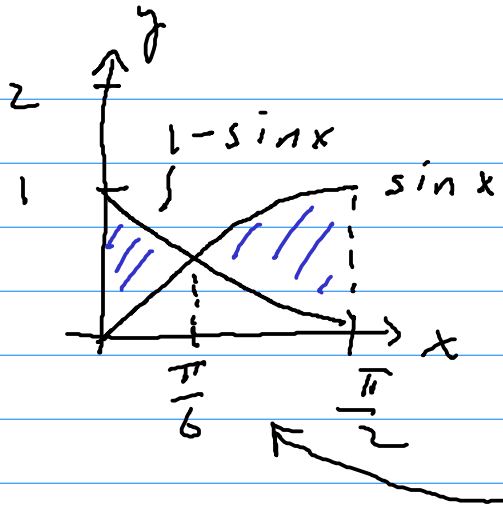
Disk Method $V = \int_0^2 \pi \left[ 6^2 - \left( -x^2 + 6 \right)^2 \right] dx$

$= \pi \int_0^2 \left( 12x^2 - x^4 \right) dx$

$= \pi \left( 4x^3 - \frac{x^5}{5} \right) \Big|_0^2$ — 6 pts

$= \pi \left( 32 - \frac{32}{5} \right)$ 1 pt

2.




$\sin x = 1 - \sin x$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ 2 pts

6 pts

$$A = \int_0^{\pi/6} (1 - \sin x - \sin x)\,dx + \int_{\pi/6}^{\pi/2} [\sin x - (1 - \sin x)]\,dx$$

$$= \int_0^{\pi/6} (1 - 2\sin x)\,dx + \int_{\pi/6}^{\pi/2} (2\sin x - 1)\,dx$$

4 pts

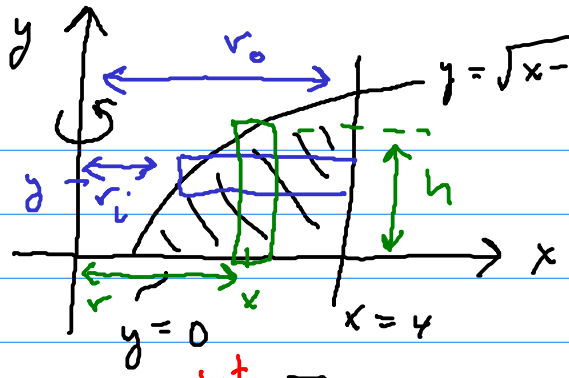
$$= (x + 2\cos x)\Big|_0^{\pi/6} + (-2\cos x - x)\Big|_{\pi/6}^{\pi/2}$$

$$= \frac{\pi}{6} + 2\cos\frac{\pi}{6} - 2 + \left[-\frac{\pi}{2} - \left(-2\cos\frac{\pi}{6} - \frac{\pi}{6}\right)\right]$$

($\cos\frac{\pi}{6} = \sqrt{3}/2$)

$$= \frac{\pi}{3} - \frac{\pi}{2} + 2\sqrt{3} - 2$$

3




, or $x = y^2 + 1$

(a) $V = \int_0^{\sqrt{3}} \pi \left[ 16 - (y^2+1)^2 \right] dy$

1 pt, 3 pt, 1 pt

(b) $V = \int_1^4 2\pi \times \sqrt{x-1}\, dx$

1 pt, 2 pt, 2 pt

4

6 pts

$$10 = \int_0^{.2} kx\,dx = k\frac{x^2}{2}\Big|_0^{.2} = k\frac{1}{2}(.2^2 - 0^2)$$

$$= k\frac{.04}{2} = k(.02)$$

$$\Rightarrow k = \frac{10}{.02} = 500$$

6 pts

$$W = \int_{-.02}^{.08} 500x\,dx = 250x^2\Big|_{-.02}^{.08}$$

$$= 250\left(.64^2 - .02^2\right) J$$

5

$$\int_1^2 4te^{3t}\,dt$$

$u = 4t \qquad v = \frac{1}{3}e^{3t}$

$u' = 4 \qquad v' = e^{3t}$

6 pt

$$= 4t\,\frac{1}{3}e^{3t}\Big|_1^2 - \int_1^2 \frac{1}{3}e^{3t}\,4\,dt \qquad 4\text{ pt}$$

$$= \frac{4}{3}te^{3t}\Big|_1^2 - \frac{4}{9}e^{3t}\Big|_1^2 \qquad 5\text{ pt}$$

$$= \frac{4}{3}\left(2e^6 - e^3\right) - \frac{4}{9}\left(e^6 - e^3\right)$$

6

$$\int \sin^5 x \cos^4 x \, dx$$

$$= \int (\sin^2 x)^2 \cos^4 x \sin x \, dx$$

$$= \int (1 - \cos^2 x)^2 \cos^4 x \sin x \, dx$$

4pt

$u = \cos x$

$du = -\sin x \, dx$

4pt

$$= \int (1 - u^2)^2 u^4 (-du)$$

$$= -\int (u^4 - 2u^6 + u^8) \, du$$

3pt

$$= -\frac{u^5}{5} + \frac{2}{7}u^7 - \frac{u^9}{9} + C$$

1pt

$$= -\frac{\cos^5 x}{5} + \frac{2}{7}\cos^7 x - \frac{\cos^9 x}{9} + C$$

7

(a) $z = 3 \sin\theta$ 3pt

(b) $x = \sqrt{5} \tan\theta$ 3pt

(c) $x = \frac{1}{\sqrt{5}} \sin\theta$ 3pt

viiiSiiiQq

8

$$\int \frac{x^3}{\sqrt{x^2-6}}\,dx$$

3pt

$x = \sqrt{6}\sec\theta$

$dx = \sqrt{6}\sec\theta\tan\theta\,d\theta$

$$= \int \frac{\sqrt{6}^3 \sec^3\theta \sqrt{6}\sec\theta\tan\theta\,d\theta}{\sqrt{6\sec^2\theta - 6}}$$

$$= \int \frac{\sqrt{6}^3 \sec^3\theta \sqrt{6}\sec\theta\tan\theta\,d\theta}{\sqrt{6}\tan\theta}$$

$$= \sqrt{6}^3 \int \sec^4\theta\,d\theta$$ 4pt

$$= \sqrt{6}^3 \int \sec^2\theta \sec^2\theta\,d\theta$$

$$= \sqrt{6}^3 \int (1+\tan^2\theta)\sec^2\theta\,d\theta$$

$$= \sqrt{6}^3 \int (1+u^2)\,du$$

$$= \sqrt{6}^3 \left(u + \frac{u^3}{3}\right) + C$$

$$= \sqrt{6}^3 \left(\tan\theta + \frac{\tan^3\theta}{3}\right) + C$$

5pt

$$= \sqrt{6}^3 \left(\frac{\sqrt{x^2-6}}{\sqrt{6}} + \frac{1}{3}\left(\frac{\sqrt{x^2-6}}{\sqrt{6}}\right)^3\right) + C$$

$\sqrt{6\sec^2\theta - 6}$

$= \sqrt{6}\sqrt{\sec^2\theta - 1}$

$= \sqrt{6}\sqrt{\tan^2\theta}$ 2pt

$= \sqrt{6}\,|\tan\theta|$

$= \sqrt{6}\tan\theta$

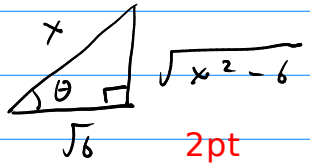
Assume $\tan\theta > 0$

$u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$x = \sqrt{6}\sec\theta$

or $\sec\theta = \frac{x}{\sqrt{6}}$




2pt