3450:335 Ordinary Differential Equations, Kreider
Worksheet for Exam 2

These problems will clarify concepts associated with undetermined coefficients and spring-mass systems. You still need to work problems to master the solution techniques. Use the text's homework and sample exams to work on the techniques.

- Identify the form of the particular solution for undetermined coefficients

$$y'' + 4y' + 3y = 2e^{3x}$$

Standard form: $y_p = Ae^{3x}$. $m = -1, -3$; no duplication so use standard form.

$$y'' + 4y' + 3y = 2e^{-3x}$$

Standard form: $y_p = Ae^{-3x}$. $m = -1, -3$; duplication so modify to $y_p = Axe^{-3x}$.

$$y'' + 2y' + 2y = e^{-x}$$

Standard form: $y_p = Ae^{-x}$. $m = -1 \pm i$; no duplication (don't be fooled just because it's the same exponential), so use standard form.

$$y'' + 2y' + 2y = e^{-x} \sin x$$

Standard form: $y_p = Ae^{-x} \sin x + Be^{-x} \cos x$. $m = -1 \pm i$; duplication so modify to $y_p = Axe^{-x} \sin x + Bxe^{-x} \cos x$.

$$y'' + 6y' + 9y = xe^{-3x}$$

Standard form $y_p = (Ax+B)e^{-3x}$. $m = -3, -3$; duplication so modify to $y_p = x^2(Ax+B)e^{-3x}$ to avoid duplication with both y_1 and y_2.

$$y'' + 9y = \sin 3x + 5 \cos 7x$$

Standard form: $y_p = A \sin 3x + B \cos 3x + C \sin 7x + D \cos 7x$. $m = \pm 3i$; duplication in first terms of y_p only, so modify to $y_p = Ax \sin 3x + Bx \cos 3x + C \sin 7x + D \cos 7x$

- Set up this problem. A 4 kg object stretches a spring 2 m. The object is removed and replaced with a 6 kg mass. The spring mass system is immersed in a liquid that imparts a damping force numerically equal to 13 times the instantaneous velocity. The mass is released from the point 2 m below equilibrium with an upward initial velocity 0.7 m/s.

  First, get k. $ks = mg$, $2k = 4 * 9.8$, so $k = 19.6$. Then $6x'' + 13x' + 19.6x = 0$ with $x(0) = +2$, $x'(0) = -0.7$.

- Convert these equations of motion into phase form. If the phase angle is in QI, use $\sin^{-1}$ or $\cos^{-1}$. If it's in QII, use $\cos^{-1}$. In QIII, find the QI reference angle and then add or subtract π. In QIV, use $\sin^{-1}$.

$$x = -2 \sin t - 4 \cos t$$
$$x = 3 \sin 5t - \cos 5t$$

First one: $A = \sqrt{2^2 + 4^2} = 4.47$. $A\cos\phi = -2$ and $A\sin\phi = -4$ so ϕ is in QIII. Ignore the signs to get the QI reference angle: $\phi_r = \sin^{-1}(4/447) = 1.1$ rad. Then add π to get $\phi = 4.25$. Finally, $x(t) = 4.47\sin(t + 4.25)$.

Second one: $A = \sqrt{3^2 + 1^2} = 3.16$. $A\cos\phi = 3$ and $A\sin\phi = -1$ so ϕ is in QIV. Then $\phi = \sin^{-1}(-1/3.16) = -0.32$. Finally, $x(t) = 3.16\sin(5t - 0.32)$.

- Identify the type of damping that generated these equations of motion

$$\begin{aligned} x &= 3e^{-t}\sin 3t - e^{-t}\cos 3t \\ x &= 6e^{-2t} + te^{-2t} \\ x &= 2e^{-t} - 7e^{-3t} \end{aligned}$$

The first is underdamping. The clue is the decaying oscillations. The second is critical damping. The clue is the t factor, implying repeated real roots and reduction of order. The third is overdampling. The clue is no oscillation.

- What is wrong with the equation of motion $x(t) = e^{-t}\sin 4t + 2e^{-3t}\cos 4t$?

The exponentials must be the same, since they come from the same characteristic equation and hence the same quadratic formula.

- Identify whether or not there is resonance in these equations of motion. If not, identify the transient and steady state components.

$$\begin{aligned} x &= 23e^{-t}\sin t - e^{-t}\cos t + \cos t \\ x &= e^{-4t} + 2te^{-4t} + \frac{1}{3}\sin 3t \\ x &= \frac{1}{4}\sin 2t - \frac{1}{8}\cos 2t + \frac{1}{6}t\cos 2t \end{aligned}$$

First one: no resonance (two clues: decaying oscillations imply friction, which is incompatible with resonance, plus there is no t times a sine or cosine).

Second one: no resonance (two clues: critical damping is incompatible with resonance, plus there is no t times a sine or cosine).

Third one: resonance (main clue is the t times the cosine, plus there is no friction [no exponential decay]).