3450:335 Ordinary Differential Equations, Kreider
Worksheet for Exam 2

These problems will clarify concepts associated with undetermined coefficients and spring-mass systems. You still need to work problems to master the solution techniques. Use the text's homework and sample exams to work on the techniques.

- Identify the form of the particular solution for undetermined coefficients

$$\begin{aligned} y'' + 4y' + 3y &= 2e^{3x} \\ y'' + 4y' + 3y &= 2e^{-3x} \\ y'' + 2y' + 2y &= e^{-x} \\ y'' + 2y' + 2y &= e^{-x}\sin x \\ y'' + 6y' + 9y &= xe^{-3x} \\ y'' + 9y &= \sin 3x + 5\cos 7x \end{aligned}$$

- Set up this problem. A 4 kg object stretches a spring 2 m. The object is removed and replaced with a 6 kg mass. The spring mass system is immersed in a liquid that imparts a damping force numerically equal to 13 times the instantaneous velocity. The mass is released from the point 2 m below equilibrium with an upward initial velocity 0.7 m/s.

- Convert these equations of motion into phase form. If the phase angle is in QI, use $\sin^{-1}$ or $\cos^{-1}$. If it's in QII, use $\cos^{-1}$. In QIII, find the QI reference angle and then add or subtract π. In QIV, use $\sin^{-1}$.

$$\begin{aligned} x &= -2\sin t - 4\cos t \\ x &= 3\sin 5t - \cos 5t \end{aligned}$$

- Identify the type of damping that generated these equations of motion

$$\begin{aligned} x &= 3e^{-t}\sin 3t - e^{-t}\cos 3t \\ x &= 6e^{-2t} + te^{-2t} \\ x &= 2e^{-t} - 7e^{-3t} \end{aligned}$$

- What is wrong with the equation of motion $x(t) = e^{-t}\sin 4t + 2e^{-3t}\cos 4t$?

- Identify whether or not there is resonance in these equations of motion. If not, identify the transient and steady state components.

$$\begin{aligned} x &= 23e^{-t}\sin t - e^{-t}\cos t + \cos t \\ x &= e^{-4t} + 2te^{-4t} + \frac{1}{3}\sin 3t \\ x &= \frac{1}{4}\sin 2t - \frac{1}{8}\cos 2t + \frac{1}{6}t\cos 2t \end{aligned}$$