3450:335 Ordinary Differential Equations, Kreider
Skills Review for Exam 3

These are some of the algebraic skills that you need for Laplace Transform problems. You should be able to do these calculations in the recommended times. You can make up your own problems to practice.

- 30 second transform the left hand side

$$\begin{aligned}
[s^2Y(s) - 6s - (-2)] + 3[sY(s) - 6] - 7[Y(s)] &= (s^2 + 3s - 7)Y(s) - 6s + 2 - 18 \\
&= (s^2 + 3s - 7)Y(s) - 6s - 16 \\
\left[s^2Y(s) - 0s - 1\right] - 4[sY(s) - 0] + 2[Y(s)] &= (s^2 - 4s + 2)Y(s) - 1 \\
\left[s^2Y(s) - 1s - 0\right] - 4[sY(s) - 1] + 2[Y(s)] &= (s^2 - 4s + 2)Y(s) - s + 4
\end{aligned}$$

- 30 second partial fraction expansions

$$\begin{aligned}
\frac{1}{s(s+4)} &= \frac{1/4}{s} - \frac{1/4}{s+4} \\
\frac{2s-3}{s(s+4)} &= \frac{-3/4}{s} + \frac{11/4}{s+4}
\end{aligned}$$

- 60 second partial fraction expansions

$$\begin{aligned}
\frac{s+2}{(s+1)(s+3)} &= \frac{1/2}{s+3} + \frac{1/2}{s+1} \\
\frac{1}{s(s+2)(s+3)} &= \frac{1/6}{s} + \frac{1/3}{s+3} - \frac{1/2}{s+2}
\end{aligned}$$

- 20 seconds: write in shifted form

$$\begin{aligned}
\frac{5s+2}{(s+1)^2+16} &= \frac{5(s+1)-3}{(s+1)^2+16} \\
\frac{2s-13}{(s+4)^2+1} &= \frac{2(s+4)-21}{(s+4)^2+1}
\end{aligned}$$

- 20 seconds: complete the square

$$\begin{aligned}
s^2 + 4s + 13 &= (s+2)^2 + 9 \\
s^2 + 6s + 13 &= (s+3)^2 + 4 \\
s^2 + 6s - 2 &= (s+3)^2 - 11
\end{aligned}$$

- 10 seconds: match to $\dfrac{2k^3}{(s^2+k^2)^2}$

  First one: $k = 2$ so $2k^3 = 16$; multiply top and bottom by 16

  Second one: $k = 3$ so $2k^3 = 54$; multiply top and bottom by 54

$$\frac{3}{(s^2+4)^2} = \left(\frac{3}{16}\right)\left(\frac{16}{(s^2+4)^2}\right)$$
$$\frac{18}{(s^2+9)^2} = \left(\frac{1}{3}\right)\left(\frac{54}{(s^2+9)^2}\right)$$

- 30 seconds: solve for $Y(s)$

$$\begin{aligned}
(sY-4)+2Y &= \frac{1}{s(s+1)} \\
Y &= \frac{4}{s+2} + \frac{1}{s(s+1)(s+2)} \\
(s^2Y+2s-3)+5(sY-2)+4Y &= 0 \\
Y &= \frac{-2s+13}{(s+1)(s+4)} \\
(s^2Y+s+1)+6Y &= \frac{1}{s} + \frac{2}{s}e^{-5s} \\
Y &= \frac{-s-1}{s^2+6} + \frac{1}{s(s^2+6)} + \frac{2}{s(s^2+6)}e^{-5s}
\end{aligned}$$

- 30 seconds: find the determinant and factor or complete the square

$$det\begin{bmatrix} s-2 & 5 \\ -2 & s-1 \end{bmatrix} = (s-2)^2+8$$
$$det\begin{bmatrix} s-3 & 1 \\ 7 & s+1 \end{bmatrix} = (s-2)^2$$

- 60 seconds: multiply and separate into components ($X(s)$ on top, $Y(s)$ on bottom)

$$\frac{1}{(s+2)(s+3)}\begin{bmatrix} 2(s-3)-12 \\ -4-3(s+7) \end{bmatrix} = \frac{1}{(s+2)(s+3)}\begin{bmatrix} 2s-18 \\ -3s-25 \end{bmatrix}$$

so $X(s) = \dfrac{2s-18}{(s+2)(s+3)}$ and $Y(s) = \dfrac{-3s-25}{(s+2)(s+3)}$. Use partial fractions to get $x(t)$ and $y(t)$.

$$\frac{1}{(s-1)^2+4}\begin{bmatrix} (s-2)+2 \\ -2+2(s-3) \end{bmatrix} = \frac{1}{(s-1)^2+4}\begin{bmatrix} s \\ 2s-8 \end{bmatrix}$$

so $X(s) = \dfrac{s}{(s-1)^2+4} = \dfrac{s-1}{(s-1)^2+4} + \dfrac{1}{(s-1)^2+4}$ and $Y(s) = \dfrac{2s-8}{(s-1)^2+4} = \dfrac{2(s-1+1)-8}{(s-1)^2+4} = \dfrac{2(s-1)}{(s-1)^2+4} - \dfrac{6}{(s-1)^2+4}$. Use formulas 18 and 19 to get $x(t)$ and $y(t)$.