3450:335 Ordinary Differential Equations, Kreider
Skills Review for Exam 2

These are some of the algebraic and integration skills that you need for second order equations. You should be able to do these calculations in the recommended times. You can make up your own problems to practice.

- 30 second factoring: find m

$$\begin{aligned} m^2+6m+13 &= 0, & m &= -3\pm 2i \\ m^2+6m+5 &= 0, & m &= -1,-5 \\ m^2-6m+5 &= 0, & m &= 1,5 \\ m^2+6m-5 &= 0, & m &= -3\pm\sqrt{14} \\ m(m-1)+6m+5 &= 0, & m &= \frac{-5}{2}\pm\frac{\sqrt{5}}{2} \\ m(m-1)-6m+5 &= 0, & m &= \frac{7}{2}\pm\frac{\sqrt{29}}{2} \\ m(m-1)+6m+6 &= 0, & m &= -2,-3 \end{aligned}$$

- 60 second Wronskians

$$\begin{aligned} W(e^{-5x}, xe^{-5x}) &= e^{-10x} \\ W(e^x\sin x, e^x\cos x) &= -e^{2x} \\ W(e^x\cos x, e^x\sin x) &= e^{2x} \\ W(e^{-x}, e^{2x}) &= 3e^x \end{aligned}$$

- 60 second eigenvalues, 120 second eigenvectors

$$\begin{aligned} A &= \begin{pmatrix} 3 & 0 \\ 8 & -1 \end{pmatrix}, & \lambda = 3, K = (1,2)'; \lambda = -1, K = (0,1)' \\ A &= \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}, & \lambda = 2, K = (1,1)' \\ A &= \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}, & \lambda_1 = i, K_1 = (1,-2-i)', \lambda_2 = -i, K_2 = (1,-2+i)' \end{aligned}$$

- 20 seconds: identify the structure of the particular solution for the right side of the ODE, assuming no duplication with the fundamental solutions

$$\begin{aligned} ay''+by'+cy &= x^2, \quad y_p = Ax^2+Bx+C \\ ay''+by'+cy &= e^{-3x}, \quad y_p = Ae^{-3x} \\ ay''+by'+cy &= xe^{2x}, \quad y_p = (Ax+B)e^{2x} \\ ay''+by'+cy &= x\sin 2x + x^2\cos 4x, \\ & y_p = (Ax+B)\sin 2x + (Cx+D)\cos 2x \\ & +(Ex^2+Fx+G)\sin 4x + (Hx^2+Ix+J)\cos 4x \\ ay''+by'+cy &= e^x\sin 3x, \quad y_p = Ae^x\sin 3x + Be^x\cos 3x \\ ay''+by'+cy &= 4x+6, \quad y_p = Ax+B \end{aligned}$$

- 42 seconds: identify the structure of the particular solution if the fundamental solutions and the right side of the ODE is given

$$\begin{aligned}
y_1 = e^x, y_2 = e^{3x} \quad g(x) &= x + 17e^{3x}, \qquad y_p = Ax + B + x\left[Ce^{ex}\right] \\
y_1 = \sin 2x, y_2 = \cos 2x \quad g(x) &= -3\sin 2x, \qquad y_p = x\left[A\sin 2x + B\cos 2x\right] \\
y_1 = e^{4x}, y_2 = xe^{4x} \quad g(x) &= 888e^{4x} + \sin x, \qquad y_p = x^2\left[Ae^{4x}\right] + B\sin x + C\cos x \\
y_1 = e^{4x}, y_2 = xe^{4x} \quad g(x) &= 888xe^{4x} + \sin 5x, \qquad y_p = x^2\left[(Ax+B)e^{4x}\right] + C\sin 5x + D\cos 5x
\end{aligned}$$

- 45 seconds: apply the initial conditions to the solution

$$\begin{array}{lll}
y = & c_1e^{-x/4} + c_2e^{-3x/5}, & y(0) = 1, y'(0) = -2 \\
y = & c_1e^{-2x} + c_2xe^{-2x}, & y(0) = -2, y'(0) = 3 \\
y = & k_1e^{-x}\sin 2x + k_2e^{-x}\cos 2x, & y(0) = 5, y'(0) = -7 \\
y = & c_1x^{-4} + c_2x^{7/5}, & y(1) = 2, y'(1) = 6 \\
y = & c_1x^{-2} + c_2x^{-2}\ln x, & y(1) = 65, y'(1) = 0 \\
y = & k_1x^2\sin(\sqrt{13}\ln x) + k_1x^2\cos(\sqrt{13}\ln x), & y(1) = 1, y'(1) = 1
\end{array}$$

First:

$$\begin{aligned}
1 &= c_1 + c_2 \\
c_1 &= 1 - c_2 \\
-2 &= -\frac{1}{4}c_1 - \frac{3}{5}c_2 \\
&= -\frac{1}{4}(1 - c_2) - \frac{3}{5}c_2 \\
&= -\frac{1}{4} + \frac{1}{4}c_2 - \frac{3}{5}c_2 \\
-\frac{7}{4} &= \frac{5-12}{20}c_2 \\
c_2 &= 5 \\
y &= -4e^{-x/4} + 5e^{-3x/5}
\end{aligned}$$

Second:

$$\begin{aligned}
-2 &= c_1 + 0 \\
3 &= -2c_1 + c_2 - 0 \\
&= 4 + c_2 \\
y &= -2e^{-2x} - xe^{-2x}
\end{aligned}$$

Third:

$$\begin{aligned}
5 &= 0 + k_2 \\
y' &= -k_1eS + 2k_1eC - k_2eC - 2k_2eS \\
-7 &= 0 + 2k_1 - k_2 - 0 \\
&= 2k_1 - 5 \\
y &= -e^{-x}\sin 2x + 5e^{-x}\cos 2x
\end{aligned}$$

Fourth:

$$\begin{aligned}
2 &= c_1 + c_2 \\
6 &= -4c_1 + \frac{7}{5}c_2 \\
&= -4(2 - c_2) + \frac{7}{5}c_2 \\
&= -8 + 4c_2 + \frac{7}{5}c_2 \\
14 &= \frac{27}{5}c_2 \\
c_2 &= \frac{70}{27} \\
c_1 &= \frac{54-70}{27} = -\frac{16}{27} \\
y &= -\frac{16}{27}x^{-4} + \frac{70}{27}x^{7/5}
\end{aligned}$$

Fifth:

$$\begin{aligned}
65 &= c_1 + 0 \\
y' &= -2c_1 - 2c_2x^{-3}\ln x + c_2x^{-2}/x \\
0 &= -2c_1 - 0 + c_2 \\
y &= 65x^{-2x} + 130x^{-2}\ln x
\end{aligned}$$

Sixth:

$$\begin{aligned}
1 &= 0 + k_2 \\
y' &= k_1 2xS + k_1x^2C \cdot \sqrt{13}/x + k_2 2xC - k_1x^2S \cdot \sqrt{13}/x \\
1 &= 0 + \sqrt{13}k_1 + 2k_2 - 0 \\
k_1 &= -1/\sqrt{13} \\
y &= -\frac{1}{\sqrt{13}}x^2\sin(\sqrt{13}\ln x) + x^2\cos(\sqrt{13}\ln x)
\end{aligned}$$