3450:335 Ordinary Differential Equations, Kreider
Skills Review for Exam 1

These are the types of algebraic and integration skills that you need for first order equations. You should be able to do these calculations in the recommended times. You can make up your own problems to practice.

- 10 second integrals

$$\int \frac{1}{u^2+1}\,du = \arctan u + c = \tan^{-1} u + c$$
$$\int \frac{u}{u^2+1}\,du = \frac{1}{2}\ln(u^2+1) + c$$
$$\int \frac{1}{4-x}\,dx = -\ln|4-x| + c$$
$$\int \frac{1}{1-4x}\,dx = -\frac{1}{4}\ln|1-4x| + c$$
$$\int e^{-4t}\,dt = -\frac{1}{4}e^{-4t} + c$$
$$\int e^{t/6}\,dt = 6e^{t/6} + c$$
$$\int e^{rt/V}\,dt = \frac{V}{r}e^{rt/V} + c$$
$$\int x^{3/5}\,dx = \frac{5}{8}x^{8/5} + c$$
$$\int ae^{ax}\,dx = e^{ax} + c$$
$$\int \cos x\,dx = \sin x + c$$
$$\int \cos x\,dy = y\cos x + c(x)$$
$$\int \cos y\,dx = x\cos y + c(y)$$
$$\int \cos y\,dy = \sin y + c$$
$$\int e^{xy}\,dx = \frac{1}{y}e^{xy} + c(y)$$
$$\int e^{xy}\,dy = \frac{1}{x}e^{xy} + c(x)$$
$$\int ye^{xy}\,dx = e^{xy} + c(y)$$
$$\int xe^{xy}\,dy = e^{xy} + c(x)$$

- 60 second integrals

$$\int xe^{ax}\,dx = \frac{1}{a}xe^{ax} - \frac{1}{a^2}e^{ax} + c$$
$$\int xe^{-ax}\,dx = -\frac{1}{a}xe^{-ax} - \frac{1}{a^2}e^{-ax} + c$$

$$\int xe^{xy}\,dx = \frac{1}{y}xe^{yx} - \frac{1}{y^2}e^{yx} + c(y)$$

$$\int x^2 \ln x\,dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + c$$

$$\int \ln x\,dx = x\ln x - x + c$$

- 5 second factoring/simplification

$$x^2y^3 + x^2 = x^2(y^3+1)$$
$$-\ln(1/5) = +\ln 5$$
$$e^{3\ln x} = x^3$$
$$e^{-2\ln x} = \frac{1}{x^2}$$
$$\frac{d}{dx}\left(e^{x^2}\right) = 2xe^{x^2}$$
$$\frac{d}{dx}\left(\ln(x^3+x+1)\right) = \frac{3x^2+1}{x^3+x+1}$$

- 15 seconds: match the form to $y' + P(x)y = Q(x)$ by identifying P and Q

$$xy' + \frac{2}{x}y = x^2e^x$$
$$e^xy' - 3y = e^{-x}$$

First: divide by x, then $P(x) = 2/x^2$. Second, divide by e^x, then $P(x) = -3e^{-x}$.

- 10 seconds: apply the product rule, assuming y is a function of x

$$\frac{d}{dx}\left(e^{x^2}y\right) = e^{x^2}y' + 2xe^{x^2}y$$
$$\frac{d}{dx}\left(x^{-5}y\right) = x^{-5}y' - \frac{5}{x^6}y$$

- 20 seconds: use the product rule to combine the terms into the form $\frac{d}{dx}(ky)$

$$\frac{y'}{x^3} - \frac{3y}{x^4} = \frac{d}{dx}\left(\frac{y}{x^3}\right)$$
$$e^{2x}y' + 2e^{2x}y = \frac{d}{dx}(e^{2x}y)$$

- 15 seconds: find u' using the chain rule, then solve for y'

$$u = y^5, \qquad u' = 5y^4y', \qquad y' = \frac{1}{5y^4}u'$$
$$u = y^{-3}, \qquad u' = -3y^{-4}y', \qquad y' = -\frac{1}{3}y^4u'$$
$$u = y^{1/4}, \qquad u' = \frac{1}{4}y^{-3/4}y', \qquad y' = 4y^{3/4}u'$$
$$u = y^{-2/3}, \qquad u' = -\frac{2}{3}y^{-5/3}y', \qquad y' = -\frac{3}{2}y^{5/3}u'$$