3450:335 Ordinary Differential Equations, Kreider

Setting up spring mass problems

1. Initial Conditions. Our coordinate system point downward, so 'down' and 'below' are positive, while 'up' and 'above' are negative.
   (a) The spring is released from the point 0.2 m below equilibrium.

   (b) The spring is released from the point 0.1 m above equilibrium with the upward velocity 2 m/s.

2. Using the equilibrium condition to find k. Sometimes, the spring constant is not given. We use the equilibrium condition $ks = mg$ to find it. Example: a 2 kg mass stretches a spring 0.4 m. Find k.

3. An object of mass 3 kg is attached to a spring with constant 12 N/m. The mass is released from the point 0.1 m above equilibrium with downward velocity 0.3 m/s. Set up and solve the ODE for the equation of motion. For reference, the solution is $x(t) = 0.15\sin(2t) - 0.1\cos(2t)$.

4. The system described above is immersed in a liquid that imparts a damping force numerically equal to 6 times the instantaneous velocity with the same initial conditions. Set up and solve the ODE for the equation of motion. For reference, the solution is $x(t) = \frac{\sqrt{3}}{15}e^{-t}\sin(\sqrt{3}t) - \frac{1}{10}e^{-t}\cos(\sqrt{3}t)$.

5. This system is moved to a new liquid that imparts a damping force numerically equal to 12 times the instantaneous velocity with the same initial conditions. Set up and solve the ODE for the equation of motion. For reference, the solution is $x(t) = -0.1e^{-2t} + 0.1te^{-2t}$.

6. This system is moved to yet another liquid with damping factor 24 times the velocity and the same initial conditions. Set up and solve the ODE for the equation of motion. For reference, the solution is $x(t) = \left(\frac{-1}{20} - \frac{\sqrt{3}}{120}\right)e^{(-4+2\sqrt{3})t} + \left(\frac{-1}{20} + \frac{\sqrt{3}}{120}\right)e^{(-4-2\sqrt{3})t}$.