consider $(1+x)y' + x^3 y = 0$

<u>without series notation</u>

1) $y' + xy' + x^3 y = 0$

2) Let $y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots$

$y' = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \dots$

so

3) $$\begin{aligned} 0 = \; & c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + \dots \\ & + c_1 x + 2c_2 x^2 + 3c_3 x^3 + 4c_4 x^4 + \dots \\ & + c_0 x^3 + c_1 x^4 + \dots \end{aligned}$$

4) we would need to write more terms to see what's happening

5) match coefficients

x^0 : $0 = c_1$

x : $0 = 2c_2 + c_1 \rightarrow c_2 = 0$

x^2 : $0 = 3c_3 + 2c_2 \rightarrow c_3 = 0$

x^3 : $0 = 4c_4 + 3c_3 + c_0 \rightarrow c_4 = -\frac{1}{4} c_0$

x^4 : $0 = 5c_5 + 4c_4 + c_1 \rightarrow c_5 = -\frac{1}{5} c_0$

$\vdots$

6) it takes a bit of effort to extend this further to see the higher order terms

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with sigma notation

1) $y' + xy' + x^3 y = 0$

2) $y = \sum_0 c_n x^n$

3) $y' = \sum_1 n c_n x^{n-1}$

4). $0 = \sum_1 n c_n x^{n-1} + \sum_1 n c_n x^n + \sum_0 c_n x^{n+3}$

$k = n-1 \qquad k = n \qquad k = n+3$

5) $0 = \sum_0 (k+1) c_{k+1} x^k + \sum_1 k c_k x^k + \sum_3 c_{k-3} x^k$

$k=0 \qquad k=1 \qquad k=2$

$= c_1 + 2c_2 x + 3c_3 x^2 + \sum_3 (k+1) c_{k+1} x^k$

$+ c_1 x + 2c_2 x^2 + \sum_3 k c_k x^k$

$+ \sum_3 c_{k-3} x^k$

$= (c_1) + (2c_2 + c_1) x + (3c_3 + 2c_2) x^2$ $\quad x^2 \quad \boxed{c_3 = 0}$

$+ \sum_3 x^k \left((k+1) c_{k+1} + k c_k + c_{k-3} \right)$

$x^0 \quad \boxed{c_1 = 0}$

$x \quad \boxed{c_2 = 0}$

all other x^k

$(k+1) c_{k+1} + k c_k + c_{k-3} = 0$

$c_{k+1} = (-k c_k - c_{k-3}) / (k+1)$

$k=3 \qquad c_4 = (-3c_3 - c_0)/4 = -c_0/4$

$k=4 \qquad c_5 = \dfrac{-4c_4 - c_1}{5} = +\frac{1}{5} c_0$

$k=5 \qquad c_6 = \dfrac{-5c_5 - c_2}{6} = -\frac{1}{6} c_0$

$\vdots$

the advantage of sigma notation is that you can get the higher coefficients quicker

$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + \cdots$

$= c_0 + 0 + 0 + 0 - \frac{1}{4} c_0 x^4 + \frac{1}{5} c_0 x^5 - \frac{1}{6} c_0 x^6$

$= c_0 \left[1 - \frac{1}{4} x^4 + \frac{1}{5} x^5 - \frac{1}{6} x^6 + \cdots \right]$