3450:335 Ordinary Differential Equations, Kreider

Second Order Problems

1. $x'' + 12x' + x = 0$

NOT complex

$m^2 + 12m + 1 = 0$

$m = \frac{-12 \pm \sqrt{144 - 4}}{2} = -6 \pm \frac{\sqrt{140}}{2}$ $\quad 140 = 35 \cdot 4$

$= -6 \pm \sqrt{35}$

$x = c_1 e^{(-6-\sqrt{35})t} + c_2 e^{(-6+\sqrt{35})t}$

$= c_1 e^{-11.9t} + c_2 e^{-.08t}$ overdamping

2. $x'' + x' + 12x = 0$

$m^2 + m + 12 = 0$

$m = \frac{-1 \pm \sqrt{1-48}}{2} = -\frac{1}{2} \pm \frac{\sqrt{47}}{2} i$

$x = c_1 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{47}}{2} t\right) + c_2 e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{47}}{2} t\right)$ underdamping

3. $x^2 y'' + 3xy' + y = 0$, $y(1) = 1$, $y'(1) = 7$

$m(m-1) + 3m + 1 = 0$

$m^2 - m + 3m + 1 = 0$

$m^2 + 2m + 1 = 0$

$m = -1, -1$

$y = c_1 x^{-1} + c_2 x^{-1} \ln x$

$= \frac{c_1}{x} + c_2 \frac{\ln x}{x}$

$1 = y(1) = c_1 + 0$

$c_1 = 1$

$y' = -\frac{c_1}{x^2} + c_2 \left[\left(-\frac{1}{x^2}\right) \ln x + \left(\frac{1}{x}\right) \frac{1}{x} \right]$

$7 = y'(1) = -c_1 + c_2 [0 + 1]$

$7 = -1 + c_2$

$c_2 = 8$

$y(x) = \frac{1}{x} + \frac{8 \ln x}{x}$

4. $x'' + 2x' + x = \sin 3t$

$m^2 + 2m + 1 = 0$

$m = -1, -1$

$x_c = c_1 e^{-t} + c_2 t e^{-t}$

transient

critical damping

$x_p = A \sin 3t + B \cos 3t$

no duplication

steady state

5. $x'' + 2x' + x = e^{2t}$

again $x_c = c_1 e^{-t} + c_2 t e^{-t}$

now $x_p = Ae^{2t}$ no duplication

6. $x'' + 2x' + x = e^{-t}$

again $x_c = c_1 e^{-t} + c_2 t e^{-t}$

now $x_p = Ae^{-t}$: duplication, so modify : $x_p = At^2 e^{-t}$

(not resonance because it's not sinusoidal)

7. $x'' + 4x = \sin t$

$m^2 + 4 = 0$

$x_c = c_1 \cos 2t + c_2 \sin 2t$

$x_p = A \sin t + B \cos t$ no duplication

8. $x'' + 4x = \sin 2t$

$x_c = c_1 \cos 2t + c_2 \sin 2t$

$x_p = A \sin 2t + B \cos 2t$: duplication, so modify :

$x_p = At \sin 2t + Bt \cos 2t$

resonance