3450:335 Ordinary Differential Equations, Kreider

Name: ______________________________

**Quiz 9**, Section 4.6, due on ______________

(10 pts) Use variation of parameters to solve $y'' - 2y' + y = e^x \tan^{-1} x$. Hint: use integration by parts to obtain both $u_1$ and $u_2$.

$m^2 - 2m + 1 = 0 \qquad (m-1)^2 = 0 \qquad y_1 = e^x \qquad y_2 = xe^x$

$m = 1, 1$

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & (e^x + xe^x) \end{vmatrix} = (e^{2x} + xe^{2x}) - xe^{2x} = e^{2x}$$

$$u_1' = -\frac{f y_2}{W} = \frac{-e^x \tan^{-1} x \cdot xe^x}{e^{2x}} = -x \tan^{-1} x$$

$u_1 = \int -x \tan^{-1} x \, dx$

$u = \tan^{-1} x \qquad du = \frac{1}{1+x^2} dx$

$dv = x\, dx \qquad v = \frac{1}{2} x^2$

$$u_1 = -\left[ \frac{1}{2} x^2 \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \right]$$

$$= -\frac{1}{2} x^2 \tan^{-1} x + \frac{1}{2} \int 1 - \frac{1}{1+x^2} dx$$

$$= -\frac{1}{2} x^2 \tan^{-1} x + \frac{x}{2} - \frac{1}{2} \tan^{-1} x$$

$$u_2' = \frac{+f y_1}{W} = \frac{e^x \tan^{-1} x \cdot e^x}{e^{2x}} = \tan^{-1} x$$

$u_2 = \int \tan^{-1} x \, dx$

$u = \tan^{-1} x \qquad du = \frac{1}{1+x^2} dx$

$dv = dx \qquad v = x$

$$u_2 = x \tan^{-1} x - \int \frac{x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \ln(1+x^2)$$

$$y_p = u_1 y_1 + u_2 y_2 = \left[ \frac{x}{2} - \frac{1}{2} \tan^{-1} x - \frac{1}{2} x^2 \tan^{-1} x \right] e^x$$

$$+ \left[ x^2 \tan^{-1} x - \frac{x}{2} \ln(1+x^2) \right] e^x$$

$$= \left[ \frac{x}{2} - \frac{1}{2} \tan^{-1} x + \frac{1}{2} x^2 \tan^{-1} x - \frac{x}{2} \ln(1+x^2) \right] e^x$$

$$y = c_1 e^x + c_2 x e^x + [\ \cdots\ ] e^x$$