

Name: _____

Quiz 8, Section 4.4, due on _____

(10 pts) Use the method of undetermined coefficients to solve the initial value problem $y'' + 6y' + 9y = (3+x)e^{-3x}$, with $y(0) = 1$, $y'(0) = 4$.

$$Y_c: m^2 + 6m + 9 = 0 \rightarrow (m+3)^2 = 0 \quad m = -3, -3$$

$$y_1 = e^{-3x} \quad y_2 = xe^{-3x}$$

Y_p : standard form $y_p = (A+Bx)e^{-3x}$
to avoid duplication, multiply by x^2

$$y_p = (Ax^2 + Bx^3)e^{-3x}$$

$$y_p' = (2Ax + 3Bx^2)e^{-3x} - 3(Ax^2 + Bx^3)e^{-3x}$$

$$= (2Ax + (3B - 3A)x^2 - 3Bx^3)e^{-3x}$$

$$y_p'' = (2A + (6B - 6A)x - 9Bx^2)e^{-3x}$$

$$- 3(2Ax + (3B - 3A)x^2 - 3Bx^3)e^{-3x}$$

$$= (2A + (6B - 12A)x + (-6B + 9A)x^2 - 9Bx^3)e^{-3x}$$

y_p''	$+ 6y_p'$	$+ 9y_p$	$=$
$2Ae^{-3x}$	$+ 0$	$+ 0$	$3e^{-3x}$
$+ (6B - 12A)x e^{-3x}$	$+ 12Ax e^{-3x}$	$+ 0$	$+ x e^{-3x}$
$- (6B + 9A)x^2 e^{-3x}$	$+ (18B - 18A)x^2 e^{-3x}$	$+ 9Ax^2 e^{-3x}$	
$+ 9Bx^3 e^{-3x}$	$- 18Bx^3 e^{-3x}$	$+ 9Bx^3 e^{-3x}$	

$$e^{-3x}: \quad 2A = 3 \quad A = \frac{3}{2} \quad x^2 e^{-3x}: \quad 0 = 0$$

$$x e^{-3x}: \quad 6B = 1 \quad B = \frac{1}{6} \quad x^3 e^{-3x}: \quad 0 = 0$$

$$y = c_1 e^{-3x} + c_2 x e^{-3x} + \frac{3}{2} x^2 e^{-3x} + \frac{1}{6} x^3 e^{-3x}$$

$$y' = -3c_1 e^{-3x} + c_2 e^{-3x} - 3c_2 x e^{-3x} + \text{stuff}$$

$$1 = y(0) = c_1 \quad 4 = y'(0) = -3c_1 + c_2, \quad c_2 = 7$$

$$y = e^{-3x} + 7x e^{-3x} + \frac{3}{2} x^2 e^{-3x} + \frac{1}{6} x^3 e^{-3x}$$