

Name: _____

Quiz 6, Section 4.1, due on _____

(10 pts) Determine if the set of functions $\{f_1(x) = \cos 2x, f_2(x) = 1, f_3(x) = \cos^2 x\}$ is linearly dependent or independent on the interval $(-\infty, \infty)$. Hint: recall that $2 \cos x \sin x$ is equivalent to $\sin 2x$.

$$W = \begin{vmatrix} \cos 2x & 1 & \cos^2 x \\ -2 \sin 2x & 0 & -\sin 2x \\ -4 \cos 2x & 0 & -2 \cos 2x \end{vmatrix} \quad \leftarrow (\cos^2 x)' = 2 \cos x \sin x = \sin 2x$$

$$= \cos 2x \begin{vmatrix} 0 & -\sin 2x \\ 0 & -2 \cos 2x \end{vmatrix} - 1 \begin{vmatrix} -2 \sin 2x & -\sin 2x \\ -4 \cos 2x & -2 \cos 2x \end{vmatrix}$$

$$+ \cos^2 x \begin{vmatrix} -2 \sin 2x & 0 \\ -4 \cos 2x & 0 \end{vmatrix}$$

$$= \cos 2x (0) - 1 (4 \sin 2x \cos 2x - 4 \sin 2x \cos 2x) + \cos^2 x (0)$$

$$= 0$$

The set is linearly dependent