3450:335 Ordinary Differential Equations, Kreider

Name: _____

Quiz 6B, Section 4.2, due on _____

(10 pts) One solution to $x^2y''(x) + xy'(x) + y(x) = 0$ is $y_1(x) = \sin(\ln x)$. Use reduction of order to find a second linearly independent solution.

$y = c_1 y_1 + c_2 y_2$

$y_2 = u y_1$

$u = \int \frac{e^{-\int p\,dx}}{y_1^2}\,dx$

standard form: $y'' + \frac{1}{x}y' + \frac{1}{x^2}y = 0$

$p = \frac{1}{x}$ $\quad e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$

$u = \int \frac{1}{x} \frac{1}{\sin^2(\ln x)}\,dx$

$r = \ln x$

$= \int \frac{1}{\sin^2(r)}\,dr = \int \csc^2 r\,dr = -\cot r$

$= -\cot(\ln x)$

$y_2 = u y_1 = -\cot(\ln x)\sin(\ln x) = -\cos(\ln x)$