

Name: _____

Quiz 4B, Section 2.5, due on _____

(10 pts) Solve the homogeneous equation $(y^2 + yx)dx + x^2 dy = 0$.

$$y = ux \quad dy = u dx + x du$$

$$(u^2 x^2 + ux^2) dx + x^2 (u dx + x du) = 0$$

$$(u^2 x^2 + 2ux^2) dx + x^3 du = 0$$

$$x^2 (u^2 + 2u) dx = -x^3 du$$

$$-\frac{1}{x} dx = \frac{1}{u+2u} du = \frac{A}{u} + \frac{B}{u+2} du = \frac{1/2}{u} - \frac{1/2}{u+2} du$$

$$-\ln x + c = \frac{1}{2} \ln u - \frac{1}{2} \ln(u+2) = \frac{1}{2} \ln \left(\frac{u}{u+2} \right)$$

$$-2 \ln x + d = \ln \left(\frac{u}{u+2} \right)$$

$$e^{-2 \ln x + d} = \frac{u}{u+2} = \frac{y/x}{y/x+2} = \frac{y}{y+2x}$$

$$e^d e^{\ln x^{-2}}$$

$$\frac{1}{k} \frac{1}{x^2} = \frac{y}{y+2x}$$

$$y+2x = kx^2 y$$

$$y - kx^2 y = -2x$$

$$y = \frac{-2x}{1 - kx^2} = \frac{2x}{kx^2 - 1}$$