3450:335 Ordinary Differential Equations, Kreider

# This is for practice only. I will not collect it. The answer key is posted on my web site

**Quiz 3B**, Section 2.4

Show that the ODE $(3x^2 + y\cos(xy) + 5x^4y^7)dx + \left(x\cos(xy) - \frac{1}{y^2+2} + 7x^5y^6 + 2\right)dy$ is exact, and find the solution.

$$M_y = 0 + [\cos(xy) - xy\sin(xy)] + 35x^4y^6$$

$$N_x = [\cos xy - yx\sin(xy)] + 35x^4y^6 \quad \checkmark$$

$$f = \int 3x^2 + y\cos(xy) + \underline{5x^4}y^7\,dx \qquad u = xy$$

$$= \underline{\underline{\cancel{3x^2}}}\ x^3 + y\left[\frac{1}{y}\sin(xy)\right] + x^5y^7 + \underline{A(y)}$$

$$\int y\cos(xy)\,dx = \int \cos(u)\,du = \sin(u)$$

$u = xy$

$du = y\,dx$

also

$$f = \int x\cos(xy) - \frac{1}{y^2+2} + \underline{7}x^5\underline{y^6} + 2\,dy \qquad u = xy$$

$$= \sin(xy) - \underline{\frac{1}{\sqrt{2}}\arctan\left(\frac{y}{\sqrt{2}}\right)} + x^5y^7 + \underline{2y} + \underline{\underline{B(x)}}$$

$$f = \sin(xy) - \frac{1}{\sqrt{2}}\arctan\left(\frac{y}{\sqrt{2}}\right) + x^5y^7 + 2y + \cancel{3x^2}\ x^3 = c$$

note: $\int \frac{1}{u^2+a^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right)$