3450:335 Ordinary Differential Equations, Kreider

Name: ____________________

Quiz 20, Section 6.1, due on ____________

(10 pts) Find the general solution to the equation $y'' + 2xy' + 2y = 0$. Write all terms up to and including the x^5 term.

$$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \cdots$$

$$y' = c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + \cdots$$

$$y'' = 2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + \cdots$$

stop at the c_5 term to get $c_5 x^5$ in the soln

$$\begin{aligned} 0 &= y'' + 2xy' + 2y \\ &= [2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + \cdots] \\ &+ [2c_1 x + 4c_2 x^2 + 6c_3 x^3 + \cdots] \\ &+ [2c_0 + 2c_1 x + 2c_2 x^2 + 2c_3 x^3 + \cdots] \end{aligned}$$

line up the powers of x for easy addition

$$0 = (2c_2 + 2c_0) + (6c_3 + 4c_1)x + (12c_4 + 6c_2)x^2 + (20c_5 + 8c_3)x^3 + \cdots$$

$$c_2 = -c_0 \qquad c_3 = -\tfrac{2}{3}c_1 \qquad c_4 = -\tfrac{1}{2}c_2 = \tfrac{1}{2}c_0 \qquad c_5 = -\tfrac{2}{5}c_3 = \tfrac{4}{15}c_1$$

$$\begin{aligned} y &= c_0 + c_1 x - c_0 x^2 - \tfrac{2}{3}c_1 x^3 + \tfrac{1}{2}c_0 x^4 + \tfrac{4}{15}c_1 x^5 + \cdots \\ &= c_0\left(1 - x^2 + \tfrac{1}{2}x^4 + \cdots\right) \\ &\quad + c_1\left(x - \tfrac{2}{3}x^3 + \tfrac{4}{15}x^5 + \cdots\right) \end{aligned}$$