3450:335 Ordinary Differential Equations, Kreider

Name: ______________________

Quiz 19, Section 7.6, due on Mon 29 July

(10 pts) Use Laplace Transforms to solve the system

$$\begin{aligned} x'(t) &= 3x + 2y \\ y'(t) &= -5x + y \end{aligned}$$

with initial conditions $x(0) = 2$, $y(0) = 1$.

$$sX - 2 = 3X + 2Y$$
$$sY - 1 = -5X + Y$$

$$\begin{bmatrix} s-3 & -2 \\ 5 & s-1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{aligned} \det &= (s-3)(s-1) - (-10) \\ &= s^2 - 4s + 3 + 10 \\ &= (s-2)^2 + 9 \end{aligned}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{\det} \begin{bmatrix} s-1 & 2 \\ -5 & s-3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{\det} \begin{bmatrix} 2s-2+2 \\ -10+s-3 \end{bmatrix}$$

$$X(s) = \frac{2s}{(s-2)^2+9} = \frac{2(s-2+2)}{(s-2)^2+9} = 2\,\frac{s-2}{(s-2)^2+9} + 4\,\frac{1}{(s-2)^2+9}\cdot\frac{3}{3}$$

$$x(t) = 2e^{2t}\cos 3t + \frac{4}{3}e^{2t}\sin 3t$$

$$Y(s) = \frac{s-13}{(s-2)^2+9} = \frac{s-2-11}{(s-2)^2+9} = \frac{s-2}{(s-2)^2+9} - 11\,\frac{1}{(s-2)^2+9}\cdot\frac{3}{3}$$

$$y(t) = e^{2t}\cos 3t - \frac{11}{3}e^{2t}\sin 3t$$