3450:335 Ordinary Differential Equations, Kreider

Name: ______________________________

Quiz 18, Section 7.5, due on ______________

(10 pts) Use Laplace Transforms to solve the initial value problem $y'' - y' = 1 + \delta(t-2)$, $y(0) = 0$, $y'(0) = 1$.

$$[s^2Y(s) - 0s - 1] - [sY(s) - 0] = \frac{1}{s} + e^{-2s}$$

keep shifted and unshifted terms separate

$$(s^2 - s)Y(s) - 1 = \frac{1}{s} + e^{-2s}$$

$$Y(s) = \frac{(1 + 1/s)}{s(s-1)} + \frac{1}{s(s-1)}e^{-2s}$$

$$= \frac{s+1}{s^2(s-1)} + \frac{1}{s(s-1)}e^{-2s}$$

- $\frac{s+1}{s^2(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} \longrightarrow s+1 = As(s-1) + B(s-1) + Cs^2$

$s=0 \quad 1 = 0 - B + 0 \qquad B = -1$

$s=1 \quad 2 = 0 + 0 + C \qquad C = 2$

$s=-1 \quad 0 = A(-1)(-2) + B(-2) + C(1)$

$0 = 2A + 2 + 2 \qquad A = -2$

- $\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1} \longrightarrow 1 = A(s-1) + Bs$

$s=1 \quad 1 = 0 + B \qquad B = 1$

$s=0 \quad 1 = -A + 0 \qquad A = -1$

- $Y(s) = \left[-\frac{2}{s} - \frac{1}{s^2} + \frac{2}{s-1}\right] + \left[-\frac{1}{s} + \frac{1}{s-1}\right]e^{-2s}$

$$y(t) = -2 - t + 2e^{t} + \left(-1 + e^{t-2}\right)\mathcal{U}(t-2)$$