3450:335 Ordinary Differential Equations, Kreider

Name: _____

Quiz 15, Section 7.2, due on _____

(10 pts) Use Laplace Transforms to solve the initial value problem $y'' + 4y = e^t$, $y(0) = 1$, $y'(0) = 0$.

$$[s^2Y(s) - 1\cdot s - 0] + 4[Y(s)] = \frac{1}{s-1}$$

$$(s^2+4)Y(s) - s = \frac{1}{s-1}$$

$$Y(s) = \frac{s}{s^2+4} + \frac{1}{(s-1)(s^2+4)}$$

$$\frac{1}{(s-1)(s^2+4)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+4}$$

$$1 = A(s^2+4) + (Bs+C)(s-1)$$

$s=1$ $\quad 1 = 5A + 0$ $\quad A = 1/5$

$s=0$ $\quad 1 = 4A + C(-1)$

$1 = 4/5 - C$ $\quad C = -1/5$

$s=-1$ $\quad 1 = 5A + (-B+C)(-2)$

$1 = 1 + 2B - 2C$

$1 = 1 + 2B + 2/5$ $\quad B = -1/5$

$$Y(s) = \frac{s}{s^2+4} + \frac{1/5}{s-1} - \frac{1}{5}\frac{s}{s^2+4} - \frac{1}{5}\frac{1}{s^2+4}\cdot\frac{2}{2}$$

$\frac{4}{5}\ \frac{s}{s^2+4}$

$$y(t) = \frac{1}{5}e^t + \frac{4}{5}\cos 2t - \frac{1}{10}\sin 2t$$