3450:335 Ordinary Differential Equations, Kreider

Name: ____________________________

Quiz 13, Section 8.2, due on ______________

(10 pts) Find the solution to the system $\bar{X}' = \begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} \bar{X}$ with $X_1(0) = 3$, $X_2(0) = 2$.

- $A - \lambda I = \begin{pmatrix} 12-\lambda & -9 \\ 4 & -\lambda \end{pmatrix}$

$$\det = (12-\lambda)(-\lambda) - (-36)$$
$$= \lambda^2 - 12\lambda + 36 = (\lambda - 6)^2$$
$$\lambda = 6, 6$$

- Find $\bar{K}$

$$(A - \lambda I)\bar{K} = \bar{0} \rightarrow \begin{pmatrix} 6 & -9 \\ 4 & -6 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$4k_1 - 6k_2 = 0 \rightarrow k_1 = \tfrac{3}{2}k_2$$

$$\bar{K} = \begin{pmatrix} 3/2\,k_2 \\ k_2 \end{pmatrix} \text{ use } k_2 = 2 \text{ so } \bar{K} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

- Find $\bar{P}$: $(A - \lambda I)\bar{P} = \bar{K}$ $\quad \begin{pmatrix} 6 & -9 \\ 4 & -6 \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$4p_1 - 6p_2 = 2$$

Let $p_2 = 0$ then $p_1 = 0$ $\quad \bar{P} = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$

- $\bar{X}(t) = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{6t} + c_2 \left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} t e^{6t} + \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} e^{6t} \right]$

- $\begin{pmatrix} 3 \\ 2 \end{pmatrix} = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$ or

$$\begin{bmatrix} 3 & 0 \\ 2 & 1/2 \end{bmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\det = 3/2 - 0$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{3/2}\begin{pmatrix} 1/2 & 0 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 3/2 + 0 \\ -6 + 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$c_1 = 1 \quad c_2 = 0$

$$\bar{X}(t) = \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{6t}$$