3450:335 Ordinary Differential Equations, Kreider

Name: _____

Quiz 11, Section 5.1, due on _____

(10 pts) Consider the spring-mass system represented by $x''(t)+16x(t) = 5\sin 4t$ with initial conditions $x(0) = 2$, $x'(0) = 1$.

$\omega^2 = 16$ $\quad M = 4$

(a) Is there damping in the system? Why or why not?

No, there is no x' term, so $\beta = 0$

(b) Is there resonance in the system? Why or why not?

yes, the natural frequency $\omega = 4$ is the same as the driving frequency $M = 4$

(c) Solve the problem.

$m^2 + 16 = 0 \rightarrow m = \pm 4i \longrightarrow x_c = c_1 \cos 4t + c_2 \sin 4t$

standard $x_p = A\sin 4t + B\cos 4t$

duplication, so adjust to $x_p = At\sin 4t + Bt\cos 4t$

DENOTE $S = \sin 4t$, $C = \cos 4t$

$x_p' = AS + 4At\,C + BC - 4Bt\,S$

$x_p'' = 4AC + (4AC - 16At\,S) - 4BS - (4BS + 16Bt\,C)$

$8AC$ $\qquad -8BS$

so

$x_p'' + 9x_p = [8AC - 16At\,S - 8BS - 16Bt\,C] + [16At\,S + 16Bt\,C]$

$= 8AC - 8BS = 5S$

$\cos 4t$: $8A = 0$

$\sin 4t$: $-8B = 5$ $\qquad B = -\frac{5}{8}$

$x = c_1\cos 4t + c_2\sin 4t - \frac{5}{8}t\cos 4t$

$x' = -4c_1\sin 4t + 4c_2\cos 4t - \frac{5}{8}\cos 4t - \frac{5}{2}t\sin 4t$

$2 = x(0) = c_1$

$1 = x'(0) = 4c_2 - \frac{5}{8}$ $\qquad \frac{13}{8} = 4c_2$ $\qquad c_2 = \frac{13}{32}$

$x = 2\cos 4t + \frac{13}{32}\sin 4t - \frac{5}{8}t\cos 4t$