3450:335 Ordinary Differential Equations, Kreider

1. Below are the equations of motion of various spring mass systems. Interpret the type of damping, and if appropriate, identify the transient and steady state components, and identify if resonance occurs.

   (a) $x(t) = 5te^{-3t} - 6e^{-3t}$

   (b) $x(t) = \frac{1}{13}e^{-3t} + \frac{1}{12}e^{-2t}$

   (c) $x(t) = e^{-2t}\sin 4t + 6e^{-2t}\cos 4t$

   (d) $x(t) = 5te^{-3t} - 6e^{-3t} + 2\sin 4t$

   (e) $x(t) = 2e^{-t}\sin t + 5e^{-t}\cos t + 6\cos 4t$

   (f) $x(t) = 2\sin 4t + 5\cos 4t + 6t\cos 4t$

2. Set up but do not solve. A 5 kg object stretches a spring 2 m. The object is replaced by a 2 kg mass, and the spring-mass system is placed in a liquid with damping factor 12. The mass is released from the point 1 m below equilibrium with initial downward velocity 2 m/s. Set up the initial value problem for the equation of motion, but do not solve.

3. Solve $y'' + 5y' + 4y = e^{-4t} + 3e^{t}$ by undetermined coefficients.

4. Solve $y'' + 15y = \sin 15t$ by undetermined coefficients.

5. Solve $y'' + 15y = \sin \sqrt{15}t$ by undetermined coefficients.

6. Solve $y'' - 2y' - 3y = 64e^{-x}$ by variation of parameters.

7. Solve $\bar{X}' = \begin{pmatrix} -4 & -3 \\ 2 & 1 \end{pmatrix} \bar{X}$ using eigenvalues and eigenvectors.

8. Solve $\bar{X}' = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} \bar{X}$ using eigenvalues and eigenvectors.