Partial Fraction Expansions

we can "plug in numbers" or "match coefficients"

- $\frac{2s-7}{(s-1)(s-3)} = \frac{A}{s-1} + \frac{B}{s-3}$

  - multiply by $(s-1)(s-3)$ to focus on numerator:

$$2s-7 = A(s-3) + B(s-1)$$

plug in numbers: $s=3$ $\quad 6-7 = 0 + 2B \quad B = -1/2$

$s=1$ $\quad 2-7 = -2A + 0 \quad A = 5/2$

so $\frac{2s-7}{(s-1)(s-3)} = \frac{5/2}{s-1} - \frac{1/2}{s-3}$

- $\frac{s+4}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$

  - multiply by $s(s^2+1)$: $\quad s+4 = A(s^2+1) + s(Bs+C)$

$s=0$ $\quad 4 = A + 0$

pick 2 other s values: $s=1$ $\quad 5 = A(2) + 1(B+C)$

$5 = 8 + B + C \rightarrow B + C = -3$

$s=-1$ $\quad 3 = A(2) - (-B+C)$

$3 = 8 + B - C \rightarrow B - C = -5$

$$\begin{array}{r} B+C=-3 \\ B-C=-5 \\ \hline 2B = -8 \end{array} \qquad B=-4 \quad \text{then} \quad (-4)+C=-3 \quad C=1$$

OR... $\quad s+4 = As^2 + A + Bs^2 + Cs$

$$0s^2 + 1s + 4 = (A+B)s^2 + Cs + A$$

$0 = A+B$

$1 = C$

$4 = A$

so $\frac{s+4}{s(s^2+1)} = \frac{4}{s} + \frac{-4s+1}{s^2+1}$