

Partial Fraction Expansions

we can "plug in numbers" or "match coefficients"

$$\bullet \frac{2s-7}{(s-1)(s-3)} = \frac{A}{s-1} + \frac{B}{s-3}$$

• multiply by $(s-1)(s-3)$ to focus on numerator:

$$2s-7 = A(s-3) + B(s-1)$$

plug in

$$\text{numbers: } s=3 \quad 6-7 = 0 + 2B \quad B = -1/2$$

$$s=1 \quad 2-7 = -2A + 0 \quad A = 5/2$$

$$\text{so } \frac{2s-7}{(s-1)(s-3)} = \frac{5/2}{s-1} - \frac{1/2}{s-3}$$

$$\bullet \frac{s+4}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

• multiply by $s(s^2+1)$: $s+4 = A(s^2+1) + s(Bs+C)$

$$s=0 \quad 4 = A + 0$$

pick 2 other
s values

$$s=1$$

$$5 = A(2) + 1(B+C)$$

$$5 = 8 + B+C \rightarrow B+C = -3$$

$$s=-1$$

$$3 = A(2) - (-B+C)$$

$$3 = 8 + B - C \rightarrow B - C = -5$$

$$B+C = -3$$

$$B-C = -5$$

$$2B = -8$$

$$B = -4$$

$$\text{then } (-4) + C = -3 \quad C = 1$$

$$\text{OR ... } s+4 = As^2 + A + Bs^2 + Cs$$

$$\underline{0s^2 + 1s + 4} = \underline{(A+B)s^2} + \underline{Cs} + \underline{A}$$

$$0 = A+B$$

$$1 = C$$

$$4 = A$$

$$\text{so } \frac{s+4}{s(s^2+1)} = \frac{4}{s} + \frac{-4s+1}{s^2+1}$$

Partial Fraction Expansion
 We can give in numbers or with coefficients

$$\frac{F-s}{(s-2)(s-1)} = \frac{A}{s-1} + \frac{B}{s-2}$$

multiply by (s-1)(s-2) to focus on numerators:

$$(s-2)A + (s-1)B = F-s$$

plug in

$s=1$	$0 + 0 = F - 1$	$F = 1$
$s=2$	$2A + 0 = F - 2$	$2A = -1$

$$\frac{F-s}{(s-2)(s-1)} = \frac{1}{s-1} - \frac{1}{2(s-2)}$$

$$\frac{F-s}{(s-2)(s-1)} = \frac{1}{s-1} + \frac{1}{2(s-2)}$$

multiply by (s-2)(s-1):

$$F-s = A(s-2) + B(s-1)$$

$s=0$	$2A = F$	$A = 1/2$
$s=1$	$0 = F - B$	$B = F = 1$

$$\frac{F-s}{(s-2)(s-1)} = \frac{1/2}{s-1} + \frac{1}{s-2}$$

$$F-s = A(s-2) + B(s-1)$$

$$F-s = (A+B)s + (-2A-B)$$

$$A+B = -1$$

$$-2A-B = F$$

$$A = 1/2$$

$$\frac{F-s}{(s-2)(s-1)} = \frac{1/2}{s-1} + \frac{1}{s-2}$$