

Exam 1 Practice Problems

These are some standard first order problems from Chapter 2 and applications from Chapter 3.

1. $(2xy^4 + 1)dx + (4x^2y^3 + 6y)dy = 0$
2. $(x^5 + 3x^2y^3)dx + (x^4y - x^3y^2)dy = 0$
3. $(y \cos x + 2xe^y) dx + (\sin x + x^2e^y - 1) dy = 0$
4. $(y^2 + 2xy)dx - x^2dy = 0$
5. $(x - y)dx + (x + y)dy = 0$
6. $\left(\frac{-2x}{(x^2 + y^2)^2} + \sin y + 2x\right) dx + \left(\frac{-2y}{(x^2 + y^2)^2} + x \cos y - 3y^2\right) dy = 0$
7. $(4x^2 + 3y^2)dx - 2xydy = 0$
8. $(y + \sqrt{x^2 - y^2})dx - xdy = 0$
9. $\left(x^3 + \frac{y}{x}\right) dx + (y^2 + \ln x)dy = 0$
10. $ye^{xy}dx + xe^{xy}dy = 0$
11. $(6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0$
12. $\left(2xy^4 - \frac{x}{\sqrt{x^2 + 1}}\right) dx + (2ye^{y^2} + 4x^2y^3) dy = 0$
13. $y^3dx + 3xy^2dy = 0$
14. $2\sqrt{x}\frac{dy}{dx} = \sqrt{1 - y^2}$
15. $y' = 1 + x + y + xy$
16. $y' = (1 - y) \cos x$
17. $3xy' + y = 12x$
18. $x^2y' = y^3 - 2xy$
19. $y^2y' = y^3 - 2xy$
20. $y^2y' + 2xy^3 = 6x$
21. $y^3\frac{dy}{dx} = (y^4 + 1) \cos x$
22. $y' + y = xe^{-x} + 1$
23. $2xy' + y^3e^{-2x} = 2xy$
24. $(x + 3)y' + (4x + 12)y = x^2 + 3x$
25. $\frac{dy}{dx} + 1 = 2y$

26. $\frac{dy}{dx} = \frac{1 + \sqrt{x}}{1 + \sqrt{y}}$

27. $e^{wT'} + e^{2T} = e^{2T-3w}$

28. $\frac{dy}{dx} + xy + \frac{x}{y^2} = 0$

29. A pie is taken from the oven when it is at 150° , and is placed outside where the air temperature is 10° . If the pie has cooled to 140° after 2 minutes, how long will it take to cool to 40° ?

30. A pond with volume 100 cubic meters has 2 polluted streams flowing into it. The first has flow rate 5 cubic meters per hour and pollution concentration 0.1 kilograms per cubic meter. The second has flow rate 7 cubic meters per hour and pollution concentration 0.2 kilograms per cubic meter. Initially, the pond has 1 kilograms of pollution in it. Assuming that the volume of the pond does not change, set up and solve the ODE for the mass of pollution in the pond.

31. On Planet K, the gravitational constant is 4 kg/s^2 and the atmospheric drag coefficient is 12 kg/s . An object with mass 6 kg is thrown down from a cliff with initial velocity $+1 \text{ m/s}$. Find the velocity of the object as a function of time.

32. Lake Zill has volume 10^6 cubic meters. A polluted river flows into it with flow rate 1000 cubic meters per day and pollution concentration 1 kilogram per cubic meter. A nearby factory dumps in pollution at the rate $500e^{-0.1t}$ kilograms per day. Initially the lake is pristine (ie, no pollution). Find the mass of pollution in the lake as a function of time.