1. $(2xy^4+1)\,dx + (4x^2y^3+6y)\,dy = 0$ exact

$M_y = 2x(4y^3)+0$

$N_x = 4(2x)y^3+0$

soln $f(x,y)=c$

$f = \int M\,dx = \int N\,dy$

$= \int 2xy^4+1\,dx = \int 4x^2y^3+6y\,dy$

$= x^2y^4 + \underline{x} + \underline{\underline{A(y)}} = x^2y^4 + \underline{\underline{3y^2}} + \underline{B(x)}$

so

$x^2y^4 + x + 3y^2 = c$

2. $(x^5+3x^2y^3)\,dx + (x^4y - x^3y^2)\,dy = 0$ homogeneous

$y = ux$

$(x^5+3x^5u^3)\,dx + (ux^5 - x^5u^2)(x\,du + u\,dx) = 0$

$(x^5+3x^5u^3+u^2x^5-x^5u^3)\,dx + (ux^6 - x^6u^2)\,du = 0$

$x^5(1+u^2+2u^3)\,dx + x^6(u-u^2)\,du = 0$

$\dfrac{u^2-u}{1+u^2+2u^3}\,du = \dfrac{1}{x}\,dx$ set up is ok, but it's too hard to integrate

3. $(y\cos x + 2xe^{y})\,dx + (\sin x + x^2e^{y} - 1)\,dy = 0$ exact

$M_y = \cos x + 2xe^{y}$

$N_x = \cos x + 2xe^{y}$

$f = \int y\cos x + 2xe^{y}\,dx = \int \sin x + x^2e^{y} - 1\,dy$

$= y\sin x + x^2e^{y} + \underline{A(y)} = y\sin x + x^2e^{y} \underline{- y} + B(x)$

so $y\sin x + x^2e^{y} - y = c$

4. $(y^2 + 2xy)\,dx - x^2\,dy = 0$ homogeneous

let $y = ux$

$(u^2x^2 + 2ux^2)\,dx - x^2(x\,du + u\,dx) = 0$

$(u^2x^2 + 2ux^2 - ux^2)\,dx - x^3\,du = 0$

$x^2(u^2+u)\,dx = x^3\,du$

$\frac{1}{x}\,dx = \frac{1}{u(u+1)}\,du = \left(\frac{1}{u} - \frac{1}{u+1}\right)du$

$\ln x + c = \ln u - \ln(u+1) = \ln\left(\frac{u}{u+1}\right)$

$kx = \frac{u}{u+1} = \frac{y/x}{y/x+1}$

$k(y+x) = \frac{y}{x} \longrightarrow kxy + x^2 = y$

$y(kx-1) = -x^2$

$y = \frac{x^2}{1-kx}$

5. $(x-y)\,dx + (x+y)\,dy = 0$ homogeneous

$y = ux$

$(x-ux)\,dx + (x+ux)(x\,du + u\,dx) = 0$

$(x-\underline{ux})\,dx + x^2\,du + ux^2\,du + (\underline{ux}+u^2x)\,dx = 0$

$(x+u^2x)\,dx + (x^2+ux^2)\,du = 0$

$x(1+u^2)\,dx = -x^2(1+u)\,du$

$-\frac{1}{x}\,dx = \frac{1+u}{1+u^2}\,du$

$-\ln x + c = \arctan(u) + \frac{1}{2}\ln(1+u^2)$

$-\ln x + c = \arctan\left(\frac{y}{x}\right) + \frac{1}{2}\ln\left(1+\frac{y^2}{x^2}\right)$

6. $\left(\frac{-2x}{(x^2+y^2)^2} + \sin y + 2x\right)dx + \left(\frac{-2y}{(x^2+y^2)^2} + x\cos y - 3y^2\right)dy = 0$

exact

$f = \int \frac{-2x}{(x^2+y^2)^2} + \sin y + 2x\,dx = \frac{1}{x^2+y^2} + x\sin y + \underline{x^2} + \underline{A(y)}$

$= \int \frac{-2y}{(x^2+y^2)^2} + x\cos y - 3y^2\,dy = \frac{1}{x^2+y^2} + x\overset{\sin}{\cancel{\cos}}y \underline{-y^3} + \underline{B(x)}$

so $\frac{1}{x^2+y^2} + x\sin y + x^2 - y^3 = c$

7. $(4x^2+3y^2)\,dx - 2xy\,dy = 0$ homogeneous

$y = ux$

$(4x^2+\underline{3u^2x^2})\,dx - 2ux^2(x\,du + \underline{u\,dx}) = 0$

$(4x^2+\underline{u^2x^2})\,dx - 2ux^3\,du = 0$

$x^2(4+u^2)\,dx = 2ux^3\,du$

$\frac{1}{x}\,dx = \frac{2u}{4+u^2}\,du$

$\ln x + c = \ln(4+u^2)$

$kx = 4+u^2 = 4+\frac{y^2}{x^2}$

$y^2 = x^2(kx-4)$

$y = \pm x\sqrt{kx-4}$

8. $(y+\sqrt{x^2-y^2})\,dx - x\,dy = 0$ homogeneous

$(\underline{ux}+\sqrt{x^2-u^2x^2})\,dx - x(x\,du + \underline{u\,dx}) = 0$

$x\sqrt{1-u^2}\,dx = x^2\,du$

$\frac{1}{x}\,dx = \frac{1}{\sqrt{1-u^2}}\,du$

$\ln x + c = \arcsin(u) = \arcsin\left(\frac{y}{x}\right)$

$x\sin(\ln x + c) = y$

9. $\left(x^3+\frac{y}{x}\right)dx + (y^2+\ln x)\,dy = 0$ exact

$f = \int x^4 + \frac{y}{x}\,dx = \int y^2 + \ln x\,dy$

$= \underline{\underline{\tfrac{1}{5}x^5}} + y\ln x + \underline{A(y)} = \underline{\tfrac{1}{3}y^3} + y\ln x + \underline{\underline{B(x)}}$

so $\frac{1}{5}x^5 + \frac{1}{3}y^3 + y\ln x = c$

10 $ye^{xy}\,dx + xe^{xy}\,dy = 0$ exact

$f = \int ye^{xy}\,dx = \int xe^{xy}\,dy$

$= y\left(\frac{1}{y}e^{xy}\right) + A(y) = x\left(\frac{1}{x}e^{xy}\right) + B(x)$

so $e^{xy} = c$ $xy = d$ $y = d/x$

11 $(6xy-y^3)\,dx + (4y+3x^2-3xy^2)\,dy = 0$ exact

$f = \int 6xy - y^3\,dx = \int 4y + 3x^2 - 3xy^2\,dy$

$= 3x^2y - xy^3 + \underline{A(y)} = \underline{2y^2} + 3x^2y - xy^3 + B(x)$

so $3x^2y - xy^3 + 2y^2 = c$

12 $(2xy^4 - \frac{x}{\sqrt{x^2+1}})\,dx + (2ye^{y^2} + 4x^2y^3)\,dy = 0$ exact

$f = \int 2xy^4 - \frac{x}{\sqrt{x^2+1}}\,dx = \int 2ye^{y^2} + 4x^2y^3\,dy$

$= x^2y^4 - \sqrt{x^2+1} + \underline{A(y)} = \underline{e^{y^2}} + x^2y^4 + B(x)$

so $x^2y^4 - \sqrt{x^2+1} + e^{y^2} = c$

13 $y^3\,dx + 3xy^2\,dy = 0$ homogeneous

$u^3x^3\,dx + 3u^2x^3(u\,dx + x\,du) = 0$

$4u^3x^3\,dx + 3u^2x^4\,du = 0$

$\frac{4}{x}\,dx = -\frac{3}{u}\,du$

$4\ln x + c = -3\ln u$

$kx^4 = \frac{1}{u^3} = \frac{x^3}{y^3}$ $\qquad y^3 = \frac{1}{kx}$ $\qquad y = \frac{1}{r}x^{-1/3}$

14 $2\sqrt{x}\frac{dy}{dx} = \sqrt{1-y^2}$ separable

$\frac{1}{\sqrt{1-y^2}}\,dy = \frac{1}{2}x^{-1/2}\,dx$

$\arcsin(y) = \sqrt{x} + c$ $\qquad y = \sin(\sqrt{x}+c)$

15 $y' = 1 + x + y + xy$ int factor $y' - (1+x)y = 1+x$

or separable:

$y' = (1+x)(1+y)$

$\frac{1}{1+y}\,dy = (1+x)\,dx$ $\qquad \ln(1+y) = x + \frac{x^2}{2} + c$

$1+y = e^{x+x^2/2+c} = ke^{x+x^2/2}$

$y = ke^{x+x^2/2} - 1$

16. $y' = (1-y)\cos x$ int factor $y' + (\cos x)y = \cos x$

or separable

$\frac{1}{1-y}\,dy = \cos x\,dx$ $\qquad -\ln(1-y) = \sin x + c$

$1-y = ke^{-\sin x}$ $\qquad y = 1-ke^{-\sin x}$

17 $3xy' + y = 12x$ int factor

$y' + \frac{1}{3x} y = 4$

$\hookrightarrow p = \frac{1}{3x} \quad k = e^{\frac{1}{3}\ln x} = x^{1/3}$

$\frac{d}{dx}(x^{\frac{1}{3}} y) = 4x^{4/3}$

$x^{\frac{1}{3}} y = 4 \frac{x^{4/3}}{4/3} + c = 3x^{4/3} + c$

$y = 3x + cx^{-1/3}$

18 $x^2 y' = y^3 - 2xy$ Bernoulli

$x^2 y' + 2xy = y^3$

$y' + \frac{2}{x} y = x^{-2} y^3 \qquad u = y^{1-3} = y^{-2}$

$u' = -2y^{-3} y' \; : \; y' = -\frac{1}{2} y^3 u'$

$\left[-\frac{1}{2} y^3 u'\right] + \frac{2}{x} y = x^{-2} y^3$

$u' - \frac{4}{x} y^{-2} = -2x^{-2}$

$u' - \frac{4}{x} u = -2x^{-2} \qquad p = -\frac{4}{x} \quad k = e^{-4\ln x} = x^{-4}$

$\frac{d}{dx}(x^{-4} u) = -2x^{-6}$

$x^{-4} u = \frac{2}{5} x^{-5} + c$

$\frac{1}{y^2} = u = \frac{2}{5} x^{-1} + cx^4$

$y^2 = \dfrac{1}{\frac{2}{5x} + cx^4}$

19 $y^2 y' = y^3 - 2xy$

$y' = y - 2xy^{-1}$ Bernoulli

$y' - y = -2xy^{-1}$ $u = y^{1-(-1)} = y^2$

$u' = 2yy'$ $y' = \frac{1}{2y}u'$

$\left[\frac{1}{2y}u'\right] - y = -2xy^{-1}$

$u' - 2y^2 = -4x$

$u' - 2u = -4x$ $P = -2$ $k = e^{-2x}$

$\frac{d}{dx}(e^{-2x}u) = -4xe^{-2x}$ int by parts

$e^{-2x}u = (1+2x)e^{-2x} + c$

$u = 1 + 2x + ce^{2x}$

$y = \pm\sqrt{1+2x+ce^{2x}}$

20 $y^2 y' + 2xy^3 = 6x$

$y' + 2xy = 6xy^{-2}$ Bernoulli $u = y^{1-(-2)} = y^3$

$u' = 3y^2 y'$ $y' = \frac{1}{3y^2}u'$

$\left[\frac{1}{3y^2}u'\right] + 2xy = 6xy^{-2}$

$u' + 6xy^3 = 18x$ $P = 6x$ $k = e^{3x^2}$

$\frac{d}{dx}(e^{3x^2}u) = 18xe^{3x^2}$

$e^{3x^2}u = 3e^{3x^2} + c$

$u = 3 + ce^{-3x^2}$

$\frac{1}{y^3} =$

$y = \left(\frac{1}{3 + ce^{-3x^2}}\right)^{1/3}$

21 $y^3 \frac{dy}{dx} = (y^4+1)\cos x$ Separable

$\frac{y^3}{y^4+1}\,dy = \cos x\,dx$

$\frac{1}{4}\ln(y^4+1) = \sin x + c$

$\ln(y^4+1) = 4\sin x + d$

$y^4+1 = ke^{4\sin x}$

$y = \pm\left(ke^{4\sin x} - 1\right)^{1/4}$

22 $y' + y = xe^{-x} + 1$ int factor

$p=1 \quad k=e^x$

$\frac{d}{dx}(e^x y) = e^x(xe^{-x}+1) = x + e^x$

$e^x y = \frac{x^2}{2} + e^x + c$

$y = \frac{x^2}{2}e^{-x} + 1 + ce^{-x}$

23 $2xy' + y^3 e^{-2x} = 2xy$ Bernoulli

$2xy' - 2xy = -e^{-2x}y^3$

$y' - y = -\frac{1}{2x}e^{-2x}y^3$ $\quad u = y^{1-3} = y^{-2}$

$u' = -2y^{-3}y' \quad y' = -\frac{1}{2}y^3u'$

$\left[-\frac{1}{2}y^3u'\right] - y = -\frac{1}{2x}e^{-2x}y^3$

$u' + 2y^{-2} = \frac{1}{x}e^{-2x}$

$u' + 2u = \frac{1}{x}e^{-2x}$ $\quad p=2 \quad k=e^{2x}$

$\frac{d}{dx}(e^{2x}u) = \frac{1}{x}$

$e^{2x}u = \ln x + c$

$u = e^{-2x}\ln x + ce^{-2x}$

$\frac{1}{y^2} =$

$y = \pm\sqrt{\frac{1}{e^{-2x}\ln x + ce^{-2x}}} = \frac{e^x}{\sqrt{\ln x + c}}$

24 $(x+3)y' + (4x+12)y = x^2+3x$ int factor.

$y' + 4y = x$

$P = 4 \quad k = e^{4x}$

$\frac{d}{dx}(e^{4x}y) = xe^{4x}$

$e^{4x}y = (\frac{1}{4}x - \frac{1}{16})e^{4x} + c$

$y = \frac{1}{4}x - \frac{1}{16} + ce^{-4x}$

25 $\frac{dy}{dx} + 1 = 2y$ int factor

$\frac{dy}{dx} - 2y = -1 \quad P = -2 \quad k = e^{-2x}$

$\frac{d}{dx}(e^{-2x}y) = -e^{-2x}$

$e^{-2x}y = \frac{1}{2}e^{-2x} + c$

$y = \frac{1}{2} + ce^{2x}$

26 $\frac{dy}{dx} = \frac{1+\sqrt{x}}{1+\sqrt{y}}$ separable

$(1+y^{1/2})\,dy = (1+x^{1/2})\,dx$

$y + \frac{2}{3}y^{3/2} = x + \frac{2}{3}x^{3/2} + c$

27 $e^w T' + e^{2T} = e^{2T-3w}$ separable

$e^w \frac{dT}{dw} = e^{2T}\cdot e^{-3w} - e^{2T} = e^{2T}(e^{-3w}-1)$

$e^{-2T}\,dT = e^{-w}(e^{-3w}-1)\,dw = (e^{-4w} - e^{-w})\,dw$

$-\frac{1}{2}e^{-2T} = -\frac{1}{4}e^{-4w} + e^{-w} + c$

$e^{-2T} = \frac{1}{2}e^{-4w} - 2e^{-w} + d$

$-2T = \ln(\frac{1}{2}e^{-4w} - 2e^{-w} + d)$

$T = -\frac{1}{2}\ln($ " $)$

28 $\frac{dy}{dx} + xy + \frac{x}{y^2} = 0$ Bernoulli

$\frac{dy}{dx} + xy = -\frac{x}{y^2}$ $u = y^{1-(-2)} = y^3$ $u' = 3y^2 y'$

$y' = \frac{1}{3y^2} u'$

$\left[\frac{1}{3y^2}u'\right] + xy = -\frac{x}{y^2}$

$u' + 3xy^3 = -3x$

$u' + 3xu = -3x$ $P = 3x$ $k = e^{\frac{3}{2}x^2}$

$\frac{d}{dx}\left(e^{\frac{3}{2}x^2}u\right) = -3xe^{\frac{3}{2}x^2}$

$e^{\frac{3}{2}x^2} u = -e^{\frac{3}{2}x^2} + c$

$u = -1 + ce^{-\frac{3}{2}x^2}$

$y = \left(-1 + ce^{-\frac{3}{2}x^2}\right)^{1/3}$

29 $T = T_m + (T_0 - T_m)e^{-kt}$ $T_m = 10$ $T_0 = 150$

$T = 10 + 140e^{-kt}$

$140 = T(2) = 10 + 140e^{-2k}$

$\frac{130}{140} = e^{-2k}$ $k = -\frac{1}{2}\ln\left(\frac{13}{14}\right) = \frac{1}{2}\ln\left(\frac{14}{13}\right) = .037$

Find t: $40 = 10 + 140e^{-kt}$

$\frac{30}{140} = e^{-kt}$ $t = -\frac{1}{k}\ln\left(\frac{3}{14}\right) = \frac{1}{k}\ln\left(\frac{14}{3}\right)$

$= 41.6$ min

30 $\frac{dP}{dt} = [r_1 c_1 + r_2 c_2] - (r_1 + r_2)\frac{P}{V}$ $r_1 = 5$, $c_1 = .1$, $r_2 = 7$, $c_2 = .2$

$P(0) = 1$

$\frac{dP}{dt} = [.5 + 1.4] - \frac{12}{100}P$

$\frac{dP}{dt} + .12P = 1.9$ $k = e^{.12t}$

$\frac{d}{dt}\left(e^{.12t}P\right) = 1.9e^{.12t}$

$e^{.12t}P = \frac{1.9}{.12}e^{.12t} + c = 15.83e^{.12t} + c$

$P = 15.83 + ce^{-.12t}$

IC $1 = P(0) = 15.83 + c$ $c = -14.83$

$P(t) = 15.83 - 14.83e^{-.12t}$

31 $m\frac{dv}{dt} = mg - kv \qquad m=6 \quad g=4 \quad k=12 \quad v(0)=1$

$$6\frac{dv}{dt} = 24 - 12v$$

$$\frac{dv}{dt} + 2v = 4 \qquad k = e^{2t}$$

$$\frac{d}{dt}\left(e^{2t}v\right) = 4e^{2t}$$

$$e^{2t}v = 2e^{2t} + c$$

$$v = 2 + ce^{-2t}$$

IC $\quad 1 = v(0) = 2 + c \qquad c = -1$

$$v(t) = 2 - e^{-2t}$$

32 $\frac{dP}{dt} = \left[rc_{in} + F\right] - r\frac{P}{V} \qquad V = 10^6 \quad r = 1000 \quad c_{in} = 1$

$P(0) = 0$

$$\frac{dP}{dt} = \left[1000 + 500e^{-.1t}\right] - .001P$$

$$\frac{dP}{dt} + .001P = 1000 + 500e^{-.1t}$$

$\hookrightarrow k = e^{.001t}$

$$\frac{d}{dt}\left(e^{.001t}P\right) = 1000e^{.001t} + 500e^{.001t}e^{-.1t}$$

$500e^{-.099t} \qquad .001 - .1 = -.099$

$$e^{.001t}P = \frac{1000}{.001}e^{.001t} + \frac{500}{-.099}e^{-.099t} + c$$

$$= 10^6 e^{\cdot\cdot} - 5051 e^{\cdot\cdot} + c$$

$$P = 10^6 - 5051e^{-.001t}e^{-.099t} + ce^{-.001t}$$

$$= 10^6 - 5051e^{-.1t} + ce^{-.001t}$$

IC $\quad 0 = P(0) = 10^6 - 5051 + c \qquad c = -994949$

$$P(t) = 10^6 - 5051e^{-.1t} - 994949e^{-.001t}$$