Applied Numerical Methods I

Cancellation error in the quadratic formula

Consider the roots of $0 = .2500x^2 + 7.325x + .1725$ in $t = 4$ digit arithmetic. The actual roots are

$$\begin{aligned} x^- &= -29.2764\cdots \\ x^+ &= -.02357\cdots \end{aligned}$$

Since b is positive, we should use the standard form for x^-:

$$x^- = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Let's approximate the roots using $t = 4$ digits. Computing $S = \sqrt{b^2 - 4ac}$ is a multi-step process:

$$\begin{aligned} b^2 : &\quad 53.655625 \to 53.66 \\ ac : &\quad .043125 \to .04313 \\ 4ac : &\quad .04313 \times 4 = .17252 \to .1725 \\ b^2 - 4ac : &\quad 53.66 - .1725 = 53.4875 \to 53.49 \\ S = \sqrt{b^2 - 4ac} : &\quad \sqrt{53.49} = 7.313685801 \to 7.314 \end{aligned}$$

Then x^- is computed as $\dfrac{-7.325 - 7.314}{.5000} = \dfrac{-14.639}{.500} \to \dfrac{-14.64}{.5000} = -29.28$, which is what we expect – we used the correct formula.

If we compute x^+ using the standard formula, we get $x^+ = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} = \dfrac{-7.325 + 7.314}{.5000} = \dfrac{-.011 * *}{.5000} = -.022 * *$. There is a loss of 2 significant digits. This is probably converted to $-.02200$, which yields a 6.7% relative error. If we use the modified formula, which is accurate, we get $x^+ = \dfrac{c}{ax^-} = \dfrac{.1725}{-7.320} = -.02356557 \to -.02357$ as expected.