Partial Pivoting Example

Consider the system below. To find the first pivot (column 1), take the element with the largest magnitude and switch rows. Here, the pivot is in row 3, so switch rows 1 and 3.

$$\left[\begin{array}{rrr|r} .7290 & .8100 & .9000 & .6867 \\ 1.000 & 1.000 & 1.000 & .8338 \\ 1.331 & 1.210 & 1.100 & 1.000 \end{array}\right] \rightarrow \left[\begin{array}{rrr|r} 1.331 & 1.210 & 1.100 & 1.000 \\ 1.000 & 1.000 & 1.000 & .8338 \\ .7290 & .8100 & .9000 & .6867 \end{array}\right]$$

The multipliers are $m_{21} = 1.000/1.331 = .7513$, $m_{31} = .7290/1.331 = .5447$. The row reduction leads to the matrix on the left, below, and pivoting requires that we switch rows 2 and 3 (because .1473 is bigger than .0909) to get the matrix on the right, below.

$$\left[\begin{array}{rrr|r} 1.331 & 1.210 & 1.100 & 1.000 \\ 0 & .0909 & .1736 & .0825 \\ 0 & .1473 & .2975 & .1390 \end{array}\right] \rightarrow \left[\begin{array}{rrr|r} 1.331 & 1.210 & 1.100 & 1.000 \\ 0 & .1473 & .2975 & .1390 \\ 0 & .0909 & .1736 & .0825 \end{array}\right]$$

Now, the multipler is $m_{32} = .0909/.1473 = .6171$, and the row reduction yields

$$\left[\begin{array}{rrr|r} 1.331 & 1.210 & 1.100 & 1.000 \\ 0 & .1473 & .2975 & .1390 \\ 0 & 0 & -.01000 & -.003280 \end{array}\right]$$

Back substitution then yields $x_3 = .3280$, $x_2 = .2812$ and $x_1 = .2246$. These values are acceptably close (in this situation of using only 4 significant digits) to the “actual” values of .3279, .2814 and .2245, respectively.