

Homework Set 9

Each problem is worth 10 points.

Due date: Tuesday 1 August

For problems 1 through 5, build a MATLAB function to evaluate the appropriate quadrature rule. Specify the integrand, limits of integration and number of subintervals (if appropriate) as inputs. For each of the problems, apply the rule to the following 2 integrals. Report answers to at least 8 digits after the decimal. Note that integral (b) gives you an indication of the degree of accuracy of each quadrature rule – if d is high enough, you'll get the exact answer.

$$(a) \int_{-3}^1 \sin(2x) dx = 0.688158561598754$$

$$(b) \int_1^4 x^3 - 3x^2 + 2 dx = 27/4 = 6.75$$

1. Implement the trapezoid rule by writing a function of the form `trap(f,a,b)`.
2. Implement Simpson's rule by writing a function of the form `simp(f,a,b)`.
3. Implement the composite trapezoid rule (use $n = 100$ subintervals for both integrals) by writing a function of the form `ctrap(f,a,b,n)`.
4. Implement the composite Simpson rule (use $n = 100$ subintervals, $m = 50$ pairs for both integrals) by writing a function of the form `csimp(f,a,b,n)`. Input the number of subintervals so it is easier to compare with other composite methods. Be careful with the n versus m notation – n is the number of subintervals, while m is the number of pairs.
5. Implement the 3 point Gaussian rule, which has nodes and weights

$$\mathbf{x} = [-\sqrt{3/5} \ 0 \ \sqrt{3/5}];$$

$$\mathbf{w} = [5/9 \ 8/9 \ 5/9];$$

by writing a function of the form `G3(f,a,b)`.

6. Consider using the composite trapezoid rule on integral (a). Use the error expression to estimate the minimum number of subintervals required for the error to be less than 10^{-9} .