3450:427/527 Applied Numerical Methods I, Kreider, Summer 2017

Homework Set 7

Each problem is worth 10 points.

Due date: Monday 24 July

1. Implement the code for Horner's algorithm discussed in class, in the form y = horner(A,r). Verify that it is working correctly by testing it on a simple function, say $P(x) = x^2 - 3x$, for which you can easily compute $P(r)$ for various $r$ values. Then use the routine to evaluate $P(x) = 3x^5 - 2x^2 + 7x - 2$ at the two points $x = 3.6$ and $x = -2.3$.

2. Consider $f(x) = \sqrt{x+1}$. (a) Find the Taylor Polynomial $T_3(x)$ centered at $x_0 = 0$. (b) Use the Taylor Remainder Theorem to estimate the maximum possible error at $x = 3.2$. (c) Use $f(3.2)$ and $T_3(3.2)$ to find the actual error at $x = 3.2$. (d) Plot $f$ and $T_3$ together on $[0, 4]$.

3. Let $f(x) = x \sin x$. Consider the interpolating polynomial $P_4(x)$ that interpolates $f$ at the points $x_0 = 0, x_1 = 0.1, x_2 = 0.2, \ldots, x_4 = 0.4$. Write the expression for the error $E_4(x)$ and use it to find the maximum possible error of interpolation in $[0, 0.4]$. Use MATLAB graphs to estimate the maximum values of $f^{(N+1)}(x)$ and $w(x) = (x - x_0) \cdots (x - x_4)$. Do not compute $P_4(x)$, just do the error analysis.

4. Implement the algorithms divdiff and newtval discussed in class.

   To use the algorithms, you'll have code similar to this:

```
v = [x vector of data]
c = [y vector of data]
D = divdiff(X,Y);
x = 0:65 ; n = length(x); y = zeros(1,n);
for k=1:n
   y(k) = newtval(D,X,x(k));
end
plot(X,Y,'o',x,y), axis([0 65 0 50])
```

   Apply the algorithms to the following problem:

   Experiments are run to measure the fuel consumption of a car travelling at a constant speed. Let $v$ be the speed in miles per hour, let $m(v)$ be the mileage in miles per gallon, and $c(v)$ be the fuel consumption in gallons per hour. The following table gives data on the fuel consumption of a 1993 Subaru Legacy (hey, it's all I could find on the internet) when run at constant speed:

| v (mi/hr) | 5 | 10 | 15 | 20 | 25 | 30 | 50 | 55 | 60 | 65 |
|---|---|---|---|---|---|---|---|---|---|---|
| c(v) (gal/hr) | .333 | .4 | .4688 | .5882 | .6757 | .75 | 1.471 | 1.447 | 1.667 | 1.912 |

Convert the fuel consumption $c(v)$ to fuel mileage $m(v)$ using the unit relation $\frac{miles}{hour} = \frac{miles}{gallon} \cdot \frac{gallons}{hour}$. Build the interpolating polynomial using divdiff for the data set $(v, m(v))$ and use it to estimate the fuel mileage when the car travels at 40 mph.

5. For 527 students. Consider $f(x) = \ln x$ on $[1, 10]$. (a) Find a set of at least 6 points in $[1, 10]$ for which the corresponding interpolating polynomial is a POOR approximation to $f(x)$. Plot $f(x)$ and the interpolating polynomial together to justify your choice of points. (b) Find a set of at most 10 points in $[1, 10]$ for which the corresponding interpolating polynomial is a GOOD approximation to $f(x)$. Plot $f(x)$ and the interpolating polynomial together to justify your choice of points.

6. For 527 students. For $f(x) = e^{2x}$, find the Pade Approximant $R_{22}(x)$. It is easiest to compute this by hand.