3450:427/527 Applied Numerical Methods I, Kreider, Summer 2017

Homework Set 4

Each problem is worth 10 points.

Due date: Thursday 6 July

1. Use MATLAB's backslash command to set up and solve the system

$$\begin{aligned} 5a - 2b + d &= 7 \\ -a + 6b + 2c - 9d &= 12 \\ 3c - 4d &= 3 \\ a + b + c + d &= 0 \end{aligned}$$

2. Use the Jacobi iteration algorithm to solve the system

$$\begin{aligned} x - 3y - z &= -8 \\ 3x + y + z &= 13 \\ 2x - y - 6z &= -3 \end{aligned}$$

   Remember that you might need to modify the system to put it into the appropriate form. Use 2 different starting guesses, $(0, 0, 0)'$ and $(10, 20, -30)'$, to see how many iterations it takes to converge. Use the error tolerance tol $= 1e-10$. Turn in a listing of the code, as well as the output. Include the number of iterations in your output.

3. Now use the Gauss-Seidel algorithm on the problem above. Use the same 2 starting guesses. Use the error tolerance tol $= 1e-10$. Turn in a listing of the code, as well as the output. Include the number of iterations in your output.

4. Let's play with the idea of ill-conditioning for a 7×7 system $Ax = B$. First, create A that is all ones except for 1.01's on the main diagonal:

```
A = ones(7,7);
for j=1:7
  A(j,j) = 1.01;
end
```

   Let $B = (1:7)'$.

   (a) Solve the system using the backslash command and call the result $x1$.

   (b) Perturb the matrix A slightly by multiplying $A(5,2)$ by 1.02 (a 2% change):

```
A(5,2) = A(5,2)*1.02
```

   Solve the system using the backslash command and call the result $x2$.

(c) Compare $x1$ and $x2$ by printing each value and the absolute value of the difference for each $j = 1 : 7$. Then check the overall effect by computing the relative error:

```
norm(x1-x2)/norm(x1)
```

Answer the question: A 2% change in one element causes a —% change in the overall solution. Is there evidence of ill-conditioning? If there is, how would you rate the severity of the problem?

(d) Now consider $Cx = B$, where B is the same, but C is a 7×7 matrix that is all ones except for 10's on the main diagonal. Solve the system and call the solution $y1$.

(e) Perturb the matrix by multiplying $C(5, 2)$ by 1.02. Solve the system and call the solution $y2$.

(f) Repeat part c. A 2% change in one element causes a —% change in the overall solution. Is there evidence of ill-conditioning? If there is, how would you rate the severity of the problem?

(g) Now compute the condition numbers of the unperturbed matrices A and C to see how they compare.

5. For 527 students. Let's check timing information on different algorithms – Gauss Elimination with Back Substitution (the routine uptrbk which is on my web site for download) and the built-in backslash command. First, build a random 1000×1000 system:

```
A = rand(1000,1000);
B = rand(1000,1);
```

To get timing information, use the cputime command:

```
z1 = cputime;
X1 = uptrbk(A,B);
z2 = cputime;
z2-z1
```

Do the same thing using the backslash command (call the solution X2) and compare the times. Clearly, the MATLAB command is much more efficient. Finally, display norm(X1-X2) to see that the 2 algorithms give essentially the same answer.

Submit a copy of your script file (but not a copy of uptrbk), the 2 CPU times and the norm. Note that uptrbk actually changes the matrix A (via row reduction), so time the backslash command first (it doesn't change A) so that both commands use the same matrix.