3450:427/527 Applied Numerical Methods I, Kreider, Summer 2017

Homework Set 3

Each problem is worth 10 points.

Due date: Thursday 29 June

For the computational problems, use the embedded function approach and publish the file as a pdf.

1. Find all the roots in $[-5, 5]$ of $f(x) = \sin x + \dfrac{1}{x^3 - 200}$ using the bisection algorithm in MATLAB with a tolerance of 10^{-10}. Plot the function first to see how many roots there are, then run the algorithm the indicated number of times to get the roots one at a time. Turn in a code listing and the answers, printed with at least 8 significant digits. Also report the number of iterations it took to find each root.

2. Use the fixed point algorithm to find the fixed point of

$$x = \frac{1}{x}\left(\sin x + \frac{1}{x}(\cos x - 1)\right)$$

   Use MATLAB to find the fixed point, which happens to lie in the interval $[0, 0.5]$. You do not need to verify that fact. Use a tolerance of 10^{-10}. Turn in the code listing and the solution, printed with at least 8 significant digits.

3. Write a Newton's Method code with the stopping criterion of (abserr $<$ tol) and (relerr $<$ tol). Set tol to 1.0e-10. Apply the code to the function below and print the root using at least 8 significant digits. Use a starting value of $x_1 = 50$.

$$f(x) \quad = x^3 + .001x^2 + 2x - .001$$

   Turn in a code listing, the answers, and the number of iterations.

4. Taken from Applied Numerical Methods with MATLAB, Chapra, 3e, 2008. The upward velocity of a rocket is

$$v = u \ln\left(\frac{m_0}{m_0 - qt}\right) - gt$$

   where v is the upward velocity (m/s), u is the velocity at which fuel is expelled relative to the rocket (m/s), m_0 is the initial mass of the rocket (kg), q is the fuel consumption rate (kg/s), and $g = 9.812$ is the gravitational constant (m/s^2). Use your bisection algorithm to determine the time t (s) it takes for the rocket to reach velocity $v = 750$ m/s when $m_0 = 160,000$ kg and $u = 1800$ m/s with $q = 2600$ kg/s. High resolution is not needed here, so let $tol = 1e - 4$. In a later assignment, you will generalize this problem by letting q vary to see how t is affected.

5. For 527 students only: Verify that Newton's Method fails for $f(x) = x^{1/3} = 0$ by showing that the condition $\left|\dfrac{f(x)f''(x)}{(f'(x))^2}\right| < 1$ is not satisfied.