3450:427/527 Applied Numerical Methods I, Kreider, Summer 2017

Homework Set 2 – Machine Representation and Computer Arithmetic

Each problem is worth 10 points.

Due date: Tuesday 20 June

1. Cancellation. Consider the computation of $y = \frac{1}{1+2x} - \frac{1-x}{1+x}$.

   a) There is no cancellation error if $x$ is not close to 1. To demonstrate this, plot $y$ in its original form for $x \in [2, 2.1]$ with increment $h = 1e - 6$. Zoom in several times on any section of the plot to see that the graph is smooth – no cancellation. Copy the zoomed in figure and paste it into a Word document.

   b) Cancellation does occur, however, if $x$ is close to 0. Plot $y$ for $x \in [-1e-6, 1e-6]$ with increment $h = 1e - 10$. You'll zoom in at two places. First, zoom in somewhere near either edge (zooming twice should reveal a roughness in the graph that is indicative of mild cancellation – paste a copy into your Word document). Second, reset the plot and zoom in at the center, $x = 0$. Zooming 3 or 4 times should show a jagged plot indicative of strong cancellation. Paste a copy of this plot, too.

   c) Rewrite the expression for $y$ algebraically and simplify as much as possible. Plot this expression as in part b, and zoom in the center, $x = 0$, 4 times to see that cancellation error does not occur. Paste a copy of this plot, too.

   Submit a copy of your code, shrunken copies of the 4 plots (1 from a, 2 from b, 1 from c), making sure that your algebraically simplified expression for $y$ appears prominently. The 4 plots should be resized so they all fit on one page.

2. Stability. Consider the recurrence relation

$$\begin{aligned} y_{n+1} &= y_n + h(y_n - 1)(2 - y_n), \\ y_1 &= 1.23, \end{aligned}$$

   where $h$ is positive. If $h$ is small enough (notate this as $0 < h \leq H$), then $y_n \to 2$ smoothly from below. If $h$ is a bit larger, though, the values of $y_n$ oscillate as $n$ increases – we say that $y_n$ overshoots and undershoots the limit $y = 2$. Write a code that identifies the threshhold value $H$ to 2 digits (for example, $h = .67$ is smooth but $h = .68$ oscillates). Start with $h = .50$ and either plot or print the $y_n$ values to see if they exceed 2.0 (this is overshoot). For plot commands, you can use

```
plot(y)
axis([0 10 1.95 2.05])
```

   to see a standardized plot. You can automate the determination of $H$ in a loop, or just try different $h$ values until you see the oscillatory behavior. Submit a copy of your code with the threshold condition clearly marked.

3. Numerical convergence. Consider the Maclauren Series $\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$, which converges for $x \in (-1, 1)$. This can be approximated by truncating the series to $-\sum_{n=1}^{N} \frac{x^n}{n}$. Write a code that computes the truncation using a range of $N$ values (put them in a loop). For $x = 0.5$ (near the center of the interval of convergence), determine the minimum $N$ value so that the truncation has 4 accurate digits (rounded) after the decimal point, and the minimum $N$ so that it has 10 accurate digits (rounded) after the decimal point. Write the truncation in the format %10.4f or %20.10f. Repeat this for $x = 0.95$, which is near the edge of the interval of convergence). We will discuss the code structure in class. Submit one copy of your code and a table with the 4 truncated values and their corresponding $N$ values.