Applied Numerical Methods I

Using Hermite interpolation to build a transition function

Suppose we have a quantity $K(t)$ that we want to model. We know the value of K for early time, for later time K, but we don't know exactly how the transition occurs. There are a variety of ways that we can model the transition; using Hermite interpolation is the first method we'll examine.

To put this into context, consider applying a coating on a piece of metal to prevent corrosion. Think of K as the mass transfer coefficient for water. A small value of K means that water can't penetrate the coating, while a larger value means it can. At first, the coating prevents water from reaching the metal. But as the coating degrades (hopefully on a time scale of decades but maybe quicker), water starts to penetrate and at some point (the precise science is unknown as yet) corrosion begins. The model problem below illustrates how we might begin to model this situation. Keep in mind that it's purely qualitative at this stage, but it does help to see how the process operates.

To put things in a simple setting, suppose we know that $K = 1$ up to time 0 and that $K = 5$ after time 2. Here we're assuming that the fun starts at $t = 0$. We'll use Hermite interpolation to build a smooth transition in the interval $(0, 2)$. Smooth in this case means that the derivatives match at $t = 0$ and $t = 2$. Our conditions are therefore

$$K(0) = 1, K'(0) = 0 \qquad K(2) = 5, K'(2) = 0$$

The modified divided difference table is

$$\begin{array}{ccccc} 0 & 1 & & & \\ 0 & 1 & 0 & & \\ 2 & 5 & A & B & \\ 2 & 5 & 0 & C & D \end{array} \tag{1}$$

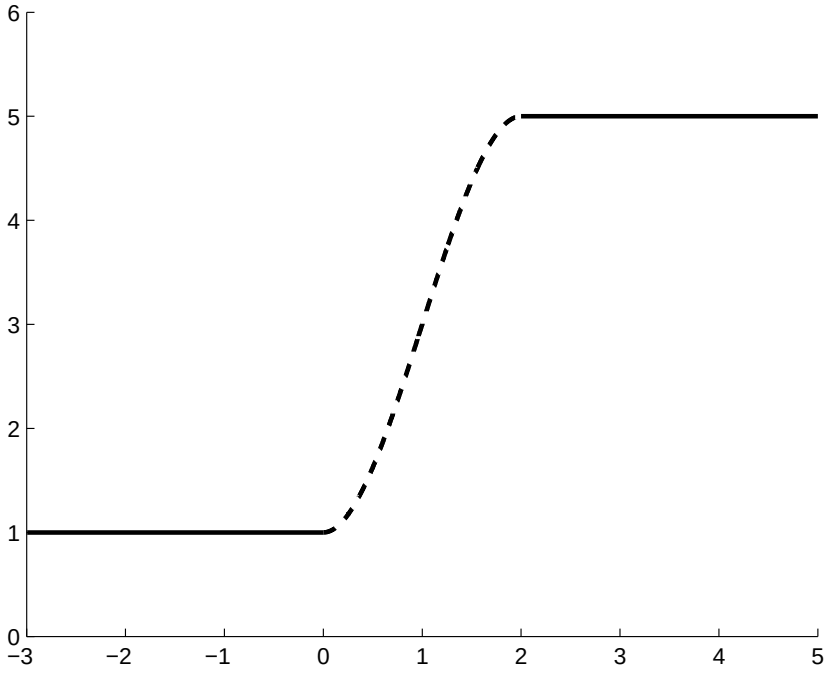
Filling in the table reveals that $A = 2$, $B = 1$, $C = -1$ and $D = -1$ so the interpolating polynomial is

$$\begin{aligned} K(t) &= 1 + 0(t-0) + 1(t-0)(t-0) - 1(t-0)(t-0)(t-1) \\ &= 1 + t^2 - t^2(t-2) \end{aligned}$$

The resulting plot is generated by

```
t1 = -3:.01:0; y1 = 1+0*t1;
t2 = 0:.001:2; y2 = 1 + t2.^2 - t2.^2.*(t2-2);
t3 = 2:.01:5; y3 = 5+0*t3;

figure(1)
hold on
plot(t1,y1,'k','Linewidth',2)
plot(t2,y2,'k--','Linewidth',2)
plot(t3,y3,'k','Linewidth',2)
axis([-3 5 0 6])
```


6
5
4
3
2
1
0
−3
−2
−1
0
1
2
3
4
5