3450:221 Calculus, Kreider

Name: ______________________________

Quiz 14, section 4.4

1. (2 pts) Evaluate $L = \lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x}$

form: $\frac{1-1}{0} = \frac{0}{0}$ so try L'H

$$L \overset{\text{L'H}}{=} \lim_{x\to 0} \frac{e^x + e^{-x}}{\cos x} = \frac{1+1}{1} = 2$$

2. (2 pts) Evaluate $L = \lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$ $= \lim_{x\to\infty} e^{\ln(1+\frac{2}{x})^x} \underset{8}{\overset{\text{x.§}}{=}} e^{x\ln(1+\frac{2}{x})} = e^k$

$k = \lim_{x\to\infty} x\ln(1+\frac{2}{x})$ form $\infty\cdot\ln 1 = \infty\cdot 0$

convert ~~to~~ fraction to apply L'H

$$= \lim_{x\to\infty} \frac{\ln(1+\frac{2}{x})}{1/x} \quad \left(\frac{\ln 1}{0} = \frac{0}{0}\right)$$

$$\overset{\text{L'H}}{=} \lim_{x\to\infty} \frac{\frac{1}{1+2/x}\cdot(-2/x^2)}{-1/x^2}$$

$$= \lim_{x\to\infty} \frac{1}{1+2/x}\cdot(+2) = \frac{1}{1+0}\cdot 2 = 2$$

So

$$L = e^k = e^2$$

3. (2 pts) Evaluate $L = \lim_{x\to\infty} \sqrt{x}e^{-x}$

$\infty\cdot 0$ so convert to a fraction

$$= \lim_{x\to\infty} \frac{\sqrt{x}}{e^x} \quad \left(\frac{\infty}{\infty}\right)$$

$$\overset{\text{L'H}}{=} \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{e^x} = \left(\frac{\frac{1}{\infty}}{\infty} : \frac{0}{\infty} = 0\right)$$

$$= 0$$

4. (2 pts) Evaluate $L = \lim_{x\to 0^+} x^x$ $\qquad 0^0$

$$L = \lim_{x\to 0^+} e^{\ln(x^x)} = \lim_{x\to 0^+} e^{x\ln x} = e^K$$

$$K = \lim_{x\to 0^+} x\ln x \qquad 0\cdot(-\infty)$$

$$= \lim_{x\to 0} \frac{\ln x}{1/x} \qquad \frac{-\infty}{\infty}$$

$$\stackrel{L'H}{=} \lim_{x\to 0} \frac{1/x}{-1/x^2}$$

$$= \lim_{x\to 0} \frac{1}{x}\cdot\frac{-x^2}{1} = \lim_{x\to 0} -x = 0$$

so $L = e^K = e^0 = 1$

5. (2 pts) Evaluate $\lim_{x\to\infty} \dfrac{e^x}{\sinh x}$ $\qquad \frac{\infty}{\infty}$

try L'H $= \lim_{x\to\infty} \dfrac{e^x}{\cosh x}$ not helpful - this is not a L'H problem.

instead $L = \lim_{x\to\infty} \dfrac{e^x}{\frac{1}{2}(e^x - e^{-x})} \qquad \dfrac{\infty}{\infty - 0}$

$$= \lim_{x\to\infty} \frac{e^x}{\frac{1}{2}(e^x - e^{-x})}\cdot\frac{e^{-x}}{e^{-x}}$$

$$= \lim_{x\to\infty} \frac{1}{\frac{1}{2}(1-e^{-2x})} = \frac{1}{\frac{1}{2}(1-0)} = 2$$