

Name: _____

Homework 1, due on Wed 29 Jan

This is practice in evaluating limits with our 3 fundamental algebraic tools - common denominator, factoring and conjugates.

1. (3 pts) Evaluate $L = \lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$. Use long division or factoring by grouping.

$$\bullet (x^3 - x^2) + (x - 1) = x^2(x - 1) + (x - 1) = (x + 1)(x - 1)$$

OR ...

expect numerator = $(x - 1)[?]$

$$\begin{array}{r} x^2 + 1 \\ x - 1 \overline{) x^3 - x^2 + x - 1} \\ \underline{-(x^3 - x^2)} \\ 0 + x - 1 \end{array}$$

$$\begin{aligned} \text{So } L &= \lim_{x \rightarrow 1} \frac{(x+1)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} \\ &= \lim_{x \rightarrow 1} \frac{x+1}{x+1} \\ &= \frac{1+1}{1+1} \\ &= 1 \end{aligned}$$

2. (4 pts) Evaluate $L = \lim_{x \rightarrow 0} \frac{5x}{\sqrt{x+16}-4} \cdot \frac{\sqrt{x+16}+4}{\sqrt{x+16}+4}$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{5x(\sqrt{x+16}+4)}{(\sqrt{x+16})^2 - (4)^2} && (x+16) - 16 = x \\ &= \lim_{x \rightarrow 0} \frac{5x(\sqrt{x+16}+4)}{x} \\ &= \lim_{x \rightarrow 0} \frac{5(\sqrt{x+16}+4)}{1} = 5(\sqrt{0+16}+4) \\ &= 5(4+4) = 40 \end{aligned}$$

3. (4 pts) Evaluate $L = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{1}{x+h} \cdot \frac{x}{x} - \frac{1}{x} \cdot \frac{x+h}{x+h} \right]$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{x - (x+h)}{x(x+h)} \right] = \lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{-h}{x(x+h)} \right) \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)} = -\frac{1}{x^2} \text{ or } -x^{-2} \end{aligned}$$