Derivatives

1 a $f(x) = \frac{1}{3} x^{-5/3} + \frac{1}{3}(x^2+1)^{-5/3}$

$f'(x) = -\frac{5}{9} x^{-8/3} - \frac{5}{9}(x^2+1)^{-8/3} \cdot 2x$

1 b $g(x) = \frac{1}{7} x^{-7} + \frac{1}{7}(\sin x + x)^{-7}$

$g'(x) = -x^{-8} - (\sin x + x)^{-8}(\cos x + 1)$

1 c $h(x) = 14x^3 + 14(e^x + \csc x)^3$

$h'(x) = 42x^2 + 42(e^x + \csc x)^2(e^x - \csc x \cot x)$

2 a $f(x) = x^2 \sin x + (3x+1)^2 \sin x + x^2 \sin(5x^2) + (3x+1)^2 \sin(5x^2)$

$f'(x) = (2x \sin x + x^2 \cos x) + (2(3x+1)3 \sin x + (3x+1)^2 \cos x)$

$+ (2x \sin(5x^2) + x^2 \cos(5x^2) 10x)$

$+ (2(3x+1)3 \sin(5x^2) + (3x+1)^2 \cos(5x^2) 10x)$

2 b $g(x) = e^x \sec x + e^x \sec(x^2) + e^x \sec(e^x)$

$g'(x) = (e^x \sec x + e^x \sec x \tan x) + (e^x \sec(x^2) + e^x \sec(x^2) \tan(x^2) 2x$

$+ (e^x \sec(e^x) + e^x \sec(e^x) \tan(e^x) e^x)$

2 c $h(x) = \frac{\tan^{-1} x}{\sin^{-1} x + 1} + \frac{\tan^{-1}(\cos x)}{\sin^{-1}(\cos x) + 1}$

$h'(x) = \frac{(\sin^{-1} x + 1)\left[\frac{1}{1+x^2}\right] - \tan^{-1}(x)\left[\frac{1}{\sqrt{1-x^2}}\right]}{(\sin^{-1} x + 1)^2}$

$+ \frac{(\sin^{-1}(\cos x) + 1)\left[\frac{-\sin x}{1+\cos^2 x}\right] - \tan^{-1}(\cos x)\left[\frac{1}{\sqrt{1-\cos^2 x}} \cdot -\sin x\right]}{(\sin^{-1}(\cos x) + 1)^2}$

2

2d

$$k(x) = x\tan^{-1}x + x^2\tan^{-1}x + x\tan^{-1}(x^2) + x^2\tan^{-1}(x^2)$$

$$k'(x) = \left(\tan^{-1}x + \frac{x}{1+x^2}\right) + \left(2x\tan^{-1}x + \frac{x^2}{1+x^2}\right)$$
$$+\left(\tan^{-1}(x^2) + \frac{x\cdot 2x}{1+x^4}\right) + \left(2x\tan^{-1}(x^2) + \frac{x^2\cdot 2x}{1+x^4}\right)$$

3a

$$f(x) = \frac{x}{x^2+1} + \frac{x}{\ln x+1} + \frac{x\ln x}{x^2+1} + \frac{x(\ln x)^2}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)[1] - x[2x]}{(x^2+1)^2} \longrightarrow \frac{1-x^2}{(x^2+1)^2}$$

$$+ \frac{(\ln x+1)[1] - x\left[\frac{1}{x}\right]}{(\ln x+1)^2} \longrightarrow \frac{\ln x}{(\ln x+1)^2}$$

$$+ \frac{(x^2+1)[\ln x+1] - x\ln x[2x]}{(x^2+1)^2} \longrightarrow \frac{\text{ok to keep as is}}{(x^2+1)^2}$$

3b

$$g(x) = \frac{\tan x}{\sec x+1} + \frac{\tan(e^x)}{\sec(e^x)+1} + \frac{\tan(\ln x)}{\sec(\ln x)+1}$$

$$g'(x) = \frac{(\sec x+1)[\sec^2 x] - \tan x[\sec x\tan x]}{(\sec x+1)^2} \longrightarrow \frac{\sec^3 x + \sec^2 x - \sec x \tan^2 x}{(\sec x+1)^2}$$

$$+ \frac{(\sec e^x+1)[\sec^2(e^x)\cdot e^x] - \tan(e^x)[\sec e^x \tan e^x \cdot e^x]}{(\sec e^x+1)^2}$$

$$+ \frac{(\sec(\ln x)+1)[\sec^2(\ln x)\frac{1}{x}] - \tan(\ln x)[\sec(\ln x)\tan(\ln x)\frac{1}{x}]}{(\sec(\ln x)+1)^2}$$

3c

$$h(x) = \frac{\tan^{-1}x}{\sin^{-1}x+1} + \frac{\tan^{-1}(\cos x)}{\sin^{-1}(\cos x)+1}$$

$$h'(x) = \frac{(\sin^{-1}x+1)\left[\frac{1}{1+x^2}\right] - \tan^{-1}x\left[\frac{1}{\sqrt{1-x^2}}\right]}{(\sin^{-1}x+1)^2}$$

$$+ \frac{(\sin^{-1}(\cos x)+1)\left[\frac{1}{1+\cos^2 x}\cdot(-\sin x)\right] - \tan^{-1}x\left[\frac{1}{\sqrt{1-\cos^2 x}}\cdot(-\sin x)\right]}{(\sin^{-1}(\cos x)+1)^2}$$

4a $f(x) = \ln(x^2) + \ln(e^x+1) + e^{\ln(x)+1} + e^{\ln(x+1)}$

Let's simplify first. $e^1 \cdot e^{\ln x}$

$f(x) = 2\ln x + \ln(e^x+1) + ex + x+1$

$f'(x) = \frac{2}{x} + \frac{e^x}{e^x+1} + e + 1$

4b $g(x) = \tan^{-1}(e^{2x}) + \sec^3(\ln x)$

$g'(x) = \frac{2e^{2x}}{1+e^{4x}} + [3\sec^2(\ln x)][\sec(\ln x)\tan(\ln x)][\frac{1}{x}]$

u^2

4c $h(x) = \sqrt{\sin^{-1}x} + \sqrt{1+(\sin^{-1}x)^2} + (1+e^{3x})^4$

$h'(x) = [\frac{1}{2}(\sin^{-1}x)^{-1/2}][\frac{1}{\sqrt{1-x^2}}]$

$+ [\frac{1}{2}(1+(\sin^{-1}x)^2)^{-1/2}][0+2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}}]$

$+ [4(1+e^{3x})^3][e^{3x} \cdot 3]$

4d $k(x) = \cos(\frac{2x}{x+1})$ numerator $= 2$

$k'(x) = [-\sin(\frac{2x}{x+1})][\frac{(x+1)[2]-2x[1]}{(x+1)^2}]$

5a $f(x) = \sin(x^2)$

$f'(x) = 2x\cos(x^2)$

$f''(x) = 2\cos(x^2) + 2x[-\sin(x^2) \cdot 2x]$

$= 2\cos(x^2) - 4x^2\sin(x^2)$

5b

$$g(x) = \frac{x^2-1}{x^2+1}$$

$$g'(x) = \frac{(x^2+1)[2x]-(x^2-1)[2x]}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

$$g''(x) = \frac{(x^2+1)^2[4]-4x[2(x^2+1)2x]}{(x^2+1)^4}$$

cancel one (x^2+1) on top and bottom

$$g''(x) = \frac{(x^2+1)[4]-4x[2\cdot 2x]}{(x^2+1)^3} = \frac{4x^2+4-16x^2}{(x^2+1)^3}$$

$$= \frac{4-12x^2}{(x^2+1)^3}$$

6a

$$3+2x^3y^4-x^2y = \frac{y}{x}$$

$$0+[6x^2]y^4+(2x^3)[4y^3y'] - \left(2xy+x^2y'\right) = \frac{y'}{x} - \frac{y}{x^2}$$

$$6x^2y^4+8x^3y^3y'-2xy-x^2y' = \frac{1}{x}y' - \frac{y}{x^2}$$

$$\left(8x^3y^3-x^2-\frac{1}{x}\right)y' = 2xy-6x^2y^4-\frac{y}{x^2}$$

$$y' = \frac{(\downarrow \quad \downarrow \quad \downarrow)}{8x^3y^3-x^2-\frac{1}{x}}$$

6b

$$e^{xy}+\tan(xy) = xy \qquad (xy)' = [1]y+x[y']$$

$$e^{xy}(y+xy')+\sec^2(xy)(y+xy') = y+xy'$$

$$ye^{xy}+xe^{xy}y'+y\sec^2(xy)+x\sec^2(xy)y' = y+xy'$$

$$\left(xe^{xy}+x\sec^2(xy)-x\right)y' = y-ye^{xy}-y\sec^2(xy)$$

$$y' = \frac{y\,(1-e^{xy}-\sec^2(xy))}{x\,(-1+e^{xy}+\sec^2(xy))} = \frac{-y}{x}$$

6c $\dfrac{xy}{x+y+1} = 166$

i) hard way (quotient) $\dfrac{(x+y+1)[y+xy'] - xy(1+y')}{(x+y+1)^2} = 0$

then $(x+y+1)(y+xy') - xy(1+y') = 0$

etc

ii) easy way: $xy = 166(x+y+1)$

$$y + xy' = 166(1+y') = 166 + 166y'$$

$$(x-166)y' = 166 - y$$

$$y' = \frac{166-y}{x-166}$$