5.5 Practice Problems

1 $I = \int 7x^3(6+x^4)^9\,dx$

$u = 6 + x^4$

$du = 4x^3\,dx \qquad x^3\,dx = \frac{1}{4}du$

$I = \int 7u^9\left(\frac{1}{4}du\right)$

$= \frac{7}{4}\left(\frac{u^{10}}{10}\right) + c = \frac{7}{40}(6+x^4)^{10} + c$

2 $I = \int x\sqrt{x+10}\,dx$

$u = x+10 \quad$ so $\quad x = u - 10$

$du = dx$

$I = \int \underline{x}\sqrt{u}\,du = \int (u-10)u^{1/2}\,du$

extra

$= \int u^{3/2} - 10u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} - 10\cdot\frac{2}{3}u^{3/2} + c$

$= \frac{2}{5}(x+10)^{5/2} - \frac{20}{3}(x+10)^{3/2} + c$

3. $I = \int \frac{1}{7}x^2\cos(2x^3-3)\,dx$

$u = 2x^3 - 3$

$du = 6x^2\,dx \qquad x^2\,dx = \frac{1}{6}du$

$I = \int \frac{1}{7}\cos(u)\left(\frac{1}{6}du\right) = \frac{1}{42}\sin(u) + c$

$= \frac{1}{42}\sin(2x^3-3) + c$

4. $I = \int \frac{6x^5}{(2x^6+137)^{15}}\,dx$

$u = 2x^6 + 137$

$du = 12x^5\,dx \qquad x^5\,dx = \frac{1}{12}du$

$I = \int 6\frac{1}{u^{15}}\left(\frac{1}{12}du\right) = \frac{1}{2}\int u^{-15}\,du$

$= \frac{1}{2}\left(\frac{u^{-14}}{-14}\right) + c$

$= -\frac{1}{28}(2x^6+137)^{-14} + c$

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5 $I = \int x(x^2+4)^{3/5}\,dx$

$u = x^2+4$

$du = 2x\,dx \qquad x\,dx = \frac{1}{2}du$

$I = \int u^{3/5}\left(\frac{1}{2}du\right) = \frac{1}{2}\left(\frac{u^{8/5}}{8/5}\right) + c$

$= \frac{5}{16}(x^2+4)^{8/5} + c$

6 $I = \int \frac{e^{2x}}{5+e^{2x}}\,dx$

$u = 5+e^{2x}$

$du = 2e^{2x}\,dx \qquad e^{2x}\,dx = \frac{1}{2}du$

$I = \int \frac{1}{u}\left(\frac{1}{2}du\right) = \frac{1}{2}\ln|u| + c$

$= \frac{1}{2}\ln(5+e^{2x}) + c$

7 $I = \int \frac{e^{2y}}{9+e^{4y}}\,dy$

$\hookrightarrow u^2 = e^{4y} \qquad u = e^{2y}$

$du = 2e^{2y}\,dy \qquad e^{2y}\,dy = \frac{1}{2}du$

$I = \int \frac{1}{9+u^2}\left(\frac{1}{2}du\right)$

$\llcorner\, a=3 \qquad I = \frac{1}{2}\left[\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)\right] + c$

$= \frac{1}{6}\tan^{-1}\left(\frac{e^{2y}}{3}\right) + c$

8 $I = \int \frac{x+3}{x+2}\,dx = \int \frac{x+2+1}{x+2}\,dx$

$= \int \frac{x+2}{x+2} + \frac{1}{x+2}\,dx = \int 1 + \frac{1}{x+2}\,dx$

$= x + \ln|x+2| + c$

9 $I = \int \frac{4x}{1+6x^4}\,dx$

$\llcorner\ u^2 = 6x^4 \qquad u = \sqrt{6}\,x^2$

$du = 2\sqrt{6}\,x\,dx$

$x\,dx = \frac{1}{2\sqrt{6}}\,du$

$I = \int 4 \cdot \frac{1}{1+u^2} \cdot \left(\frac{1}{2\sqrt{6}}\,du\right)$

$= \frac{2}{\sqrt{6}} \tan^{-1}(u) + c$

$= \frac{2}{\sqrt{6}} \tan^{-1}(\sqrt{6}\,x^2) + c$

10 $I = \int \frac{x}{9+x^4}\,dx$

$\llcorner\ u^2 = x^4 \qquad u = x^2$

$du = 2x\,dx \qquad x\,dx = \frac{1}{2}\,du$

$I = \int \frac{1}{9+u^2}\left(\frac{1}{2}\,du\right)$

$\llcorner\ a = 3 \qquad = \frac{1}{2}\left[\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)\right] + c$

$= \frac{1}{6}\tan^{-1}\left(\frac{x^2}{3}\right) + c$

12 $I = \int \frac{1}{x}\csc(\ln x)\cot(\ln x)\,dx$

$u = \ln x \quad du = \frac{1}{x}\,dx$

$I = \int \csc(u)\cot(u)\,du$

$= -\csc(u) + c \qquad = -\csc(\ln x) + c$

11 $I = \int \frac{1}{9+5x^2}\, dx$

(i) if your target is $\int \frac{1}{1+u^2}\, du$:

$I = \frac{1}{9} \int \frac{1}{1+\left(\frac{5x^2}{9}\right)}\, dx$

$u^2 = \frac{5x^2}{9} \qquad u = \frac{\sqrt{5}}{3}\, x$

$du = \frac{\sqrt{5}}{3}\, dx \qquad dx = \frac{3}{\sqrt{5}}\, du$

$I = \frac{1}{9} \int \frac{1}{1+u^2} \left(\frac{3}{\sqrt{5}}\right) du$

$= \frac{1}{3\sqrt{5}} \tan^{-1}(u) + c$

$= \frac{1}{3\sqrt{5}} \tan^{-1}\left(\frac{\sqrt{5}}{3}x\right) + c$

(ii) if your target is $\int \frac{1}{a^2+u^2}\, du$:

$I = \int \frac{1}{5\left(\frac{9}{5}+x^2\right)}\, dx = \frac{1}{5} \int \frac{1}{9/5 + x^2}\, dx$

$a^2 = 9/5, \quad a = 3/\sqrt{5}$

$I = \frac{1}{5}\left[\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)\right] + c$

$= \frac{1}{5 \cdot \frac{3}{\sqrt{5}}} \tan^{-1}\left(\frac{x}{3/\sqrt{5}}\right) + c$

$= \frac{1}{3\sqrt{5}} \tan^{-1}\left(\frac{\sqrt{5}}{3}x\right) + c$