5.5 I Definite Integrals

In a integral $\int_a^b f'(u(x))\,dx$, when we convert x to u, we must also convert the limits of integration from x values to u values

Suppose $u = x^2 - 2$ for $\int_3^7 \sim dx$

The lower limit $x=3$ goes to $u = (3)^2 - 2 = 7$

and $x = 7$ $u = 7^2 - 2 = 47$

so the converted integral would be $\int_7^{47} \sim du$

Graphical View

Consider $I = \int_0^4 \sqrt{3x+4}\,dx$

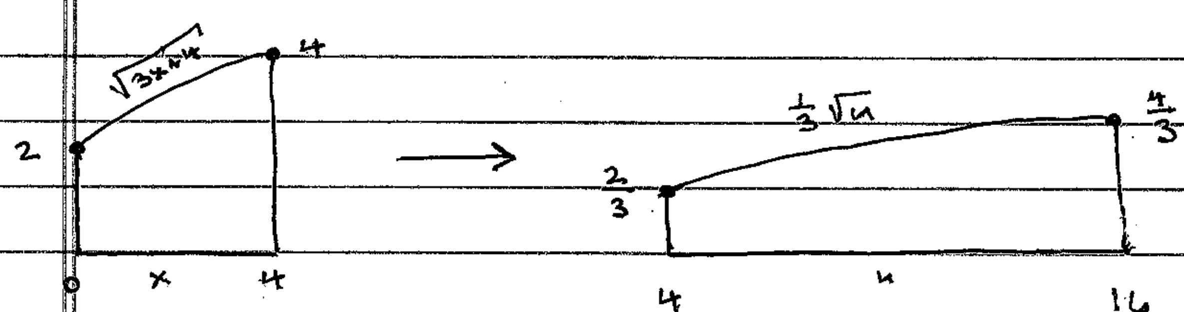
$u = 3x+4$, $du = 3\,dx$ $dx = \frac{1}{3}\,du$

lower: when $x=0$, $u=4$

upper: when $x=4$, $u=16$

so

$I = \int_4^{16} \frac{1}{3}\sqrt{u}\,du$



3 times as wide

but $\frac{1}{3}$ as tall,

so area stays the same

5.5 I 2

Ex 1 $I = \int_1^2 2x(x^2+1)^{3/4}\,dx$

$u = x^2+1 \qquad du = 2x\,dx$

$x=1 \rightarrow u=2$

$x=2 \rightarrow u=5$

$I = \int_2^5 u^{3/4}\,du = \frac{4}{7}u^{7/4}\Big|_2^5 = \frac{4}{7}\left(5^{7/4} - 2^{7/4}\right)$

Ex 2 $I = \int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$

$u = 1+\sqrt{y} \qquad du = \frac{1}{2\sqrt{y}}\,dy$

$y=1 \rightarrow u=2$

$y=4 \rightarrow u=3$

$I = \int_2^3 \frac{1}{u^2}\,du = -\frac{1}{u}\Big|_2^3 = \left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)$

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Ex 3 $I = \int_{\ln 2}^{\ln 10} \frac{e^x}{1+e^{2x}}\,dx$

$u = e^x \qquad du = e^x\,dx$

$x=\ln 2 \rightarrow u=2$

$x=\ln 10 \rightarrow u=10$

$I = \int_2^{10} \frac{1}{1+u^2}\,du = \tan^{-1} u\Big|_2^{10} = \tan^{-1} 10 - \tan^{-1} 2$