5.5 H $\int \frac{1}{u}\,du$ vs $\int \frac{1}{1+u^2}\,du$

The distinction lies with the numerator in

$$\int \frac{u'(x)}{\text{denom}}\,dx$$

Is the numerator essentially the derivative of the denominator (ln|u| form) or not ?

Pair 1 $I = \int \frac{e^x}{1+e^x}\,dx$ vs $J = \int \frac{e^x}{1+e^{2x}}\,dx$

$\uparrow$ $\uparrow$

num = e^x is deriv denom — num is deriv of e^x

$u = 1+e^x \quad du = e^x\,dx$ — $u = e^x \quad du = e^x\,dx$

$I = \int \frac{1}{u}\,du = \ln|e^x+1| + c$ — $J = \int \frac{1}{1+u^2}\,du = \tan^{-1}(e^x) + c$

Pair 2 $I = \int \frac{e^{2x}}{1+e^{2x}}\,dx$ — $J = \int \frac{e^{2x}}{1+e^{4x}}\,dx$

$u = 1+e^{2x}$ — $u = e^{2x}$

$du = 2e^{2x}\,dx$ — $du = 2e^{2x}\,dx$

$I = \int \frac{1}{u}\left(\frac{1}{2}du\right)$ — $J = \int \frac{1}{1+u^2}\left(\frac{1}{2}du\right)$

$= \frac{1}{2}\ln|1+e^{2x}| + c$ — $= \frac{1}{2}\tan^{-1}(e^{2x}) + c$

5.5 H 2

Pair 3 $\quad I = \int \frac{\cos x}{1+\sin x}\,dx$ $\qquad J = \int \frac{\cos x}{1+\sin^2 x}\,dx$

$u = 1+\sin x \quad du = \cos x\,dx$ $\qquad u = \sin x \quad du = \cos x\,dx$

$I = \int \frac{1}{u}\,du = \ln|1+\sin x| + c$ $\qquad J = \int \frac{1}{1+u^2}\,du = \tan^{-1}(\sin x) + c$

- If the denominator is $1+x^4$, what numerator gives $\int \frac{1}{u}\,du$? $\quad u = 1+x^4$

$\int \frac{1}{1+u^2}\,du \quad u = x^2$

$I = \int \frac{4x^3}{1+x^4}\,dx$

$u = 1+x^4 \quad du = 4x^3\,dx$

$I = \int \frac{1}{u}\,du = \ln|1+x^4| + c$

$J = \int \frac{2x}{1+x^4}\,dx \qquad u = x^2 \quad du = 2x\,dx$

$= \int \frac{1}{1+u^2}\,du$

$= \tan^{-1}(x^2)\,dx$

Pair 4 $\quad I = \int \frac{1}{x(1+\ln x)}\,dx$ vs $J = \int \frac{1}{x(1+\ln^2 x)}\,dx$

$u = 1+\ln x \quad du = \frac{1}{x}\,dx$ $\qquad u = \ln x \quad du = \frac{1}{x}\,dx$

$I = \int \frac{1}{u}\,du$ $\qquad J = \int \frac{1}{1+u^2}\,du$

$= \ln|1+\ln x| + c$ $\qquad = \tan^{-1}(\ln x) + c$

5.5 H 3

Pair 5 $\quad I = \int \frac{x^{1/2}}{1+x^{3/2}}\,dx$ vs $J = \int \frac{x^{-1/4}}{1+x^{3/2}}\,dx$

- I. Note that $x^{1/2}$ is essentially the derivative of $1+x^{3/2}$:

$$u = 1+x^{3/2}, \quad du = \tfrac{3}{2}x^{1/2}\,dx, \quad x^{1/2}\,dx = \tfrac{2}{3}\,du$$

$$I = \int \frac{1}{u}\left(\tfrac{2}{3}\,du\right) = \tfrac{2}{3}\ln\left|1+x^{3/2}\right| + C$$

- J. Note that $x^{-1/4}$ is NOT the derivative of $1+x^{3/2}$, so the only option is that the denominator is $1+u^2$

$$u^2 = x^{3/2}, \quad u = x^{3/4}, \quad du = \tfrac{3}{4}x^{-1/4}\,dx, \quad x^{-1/4}\,dx = \tfrac{4}{3}\,du$$

$$J = \int \frac{1}{1+u^2}\left(\tfrac{4}{3}\right)du = \tfrac{4}{3}\tan^{-1}\left(x^{3/4}\right) + C$$