5.5 E Trig functions

Template: $\int \sin u\,du = -\cos u + C$

$\int \cos u\,du = \sin u + C$

$\int \sec^2 u\,du = \tan u + C$

$\int \csc^2 u\,du = -\cot u + C$

$\int \sec u \tan u\,du = \sec u + C$

$\int \csc u \cot u\,du = -\csc u + C$

1. $I = \int \sin(2x)\,dx \qquad u = 2x, \quad du = 2\,dx, \quad dx = \frac{1}{2}du$

$I = \int \sin u \left(\frac{1}{2}du\right) = -\frac{\cos(u)}{2} + C = -\frac{1}{2}\cos(2x) + C$

2. $I = \int x\cos(x^2)\,dx$

$u = x^2 \quad du = 2x\,dx, \qquad x\,dx = \frac{1}{2}du$

$I = \int \cos u \left(\frac{1}{2}du\right) = \frac{1}{2}\sin(u) + C = \frac{1}{2}\sin(x^2) + C$

3. $I = \int \sec(4x)\tan(4x)\,dx$

$u = 4x \quad du = 4\,dx \quad dx = \frac{1}{4}du$

$I = \int \sec u \tan u \left(\frac{1}{4}du\right) = \frac{1}{4}\sec(u) + C = \frac{1}{4}\sec(4x) + C$

4. $I = \int e^{3x}\sec(e^{3x})\tan(e^{3x})\,dx$

$u = e^{3x} \quad du = 3e^{3x}dx \quad e^{3x}dx = \frac{1}{3}du$

$I = \int \sec(u)\tan(u)\left(\frac{1}{3}du\right) = \frac{1}{3}\sec(u) + C$

$= \frac{1}{3}\sec(e^{3x}) + C$

5.5 ≡ 2

5 $I = \int 6x^2 \sec^2(x^3)\,dx$

$u = x^3 \quad du = 3x^2\,dx$

$x^2\,dx = \frac{1}{3}\,du$

$I = \int 6\sec^2(u)\left(\frac{1}{3}\,du\right)$

$= 2\tan(u) + c = 2\tan(x^3) + c$

6 $I = \int \cos x \sin^3 x\,dx$

$(\sin x)^3$ so $u = \sin x, \quad du = \cos x\,dx$

$I = \int u^3\,du = \frac{1}{4}\sin^4 x + c$

7 $I = \int \frac{\sin(\sqrt{x}+1)}{\sqrt{x}}\,dx$

$u = \sqrt{x}+1 \quad du = \frac{1}{2\sqrt{x}}\,dx \quad \frac{1}{\sqrt{x}}\,dx = 2\,du$

$I = \int \sin(u)\,(2\,du) = -2\cos(u) + c$

$= -2\cos(\sqrt{x}+1) + c$

8 $I = \int \csc^2\left(\frac{x}{6}\right)dx$

$u = \frac{x}{6} \quad du = \frac{1}{6}\,dx, \quad dx = 6\,du$

$I = \int \csc^2 u\,(6\,du) = -6\cot u + c = -6\cot\left(\frac{x}{6}\right) + c$

5.5 E 3

9 $I = \int e^x \cos(e^x)\, dx$

$u = e^x \quad du = e^x\, dx$

$I = \int \cos u\, du = \sin u + c = \sin(e^x) + c$

10 $I = \int 2x\cos(x^2) + 2e^{2x}\cos(e^{2x})\, dx$

$u = x^2 \qquad v = e^{2x}$

$du = 2x\, dx \qquad dv = 2e^{2x}\, dx$

$I = \int \cos(u)\, du + \cos(v)\, dv$

$= \int \cos(u)\, du + \int \cos(v)\, dv$

$= \sin(x^2) + \sin(e^{2x}) + c$