5.5 D Powers and Roots

Here, the shell is u^n

The template is $\int u^n \, du = \dfrac{u^{n+1}}{n+1} + c$

1. $I = \int 2x\,(x^2-3)^4 \, dx$

$\leftarrow \quad u = x^2 - 3$

$du = 2x\,dx$

$I = \int (x^2-3)^4 \, 2x\,dx$

$= \int u^4 \, du = \dfrac{u^5}{5} + c = \dfrac{(x^2-3)^5}{5} + c$

2. $I = \int \dfrac{e^x}{(1+e^x)^2} \, dx$

$u = 1 + e^x \qquad du = e^x \, dx$

$I = \int \frac{1}{u^2}\,du = \int \frac{du}{u^2} = \int u^{-2}\,du$

$= \dfrac{u^{-1}}{-1} + c = -\dfrac{1}{u} + c = \dfrac{-1}{1+e^x} + c$

3. $I = \int \cos x \, (4+\sin x)^{3/5} \, dx$

$u = 4 + \sin x \qquad du = \cos x \, dx$

$I = \int (4+\sin x)^{3/5} \cos x \, dx$

$= \int u^{3/5} \, du = \frac{5}{8} u^{8/5} + c = \frac{5}{8}(4+\sin x)^{3/5} + c$

4. $I = \int \dfrac{1}{x(2+\ln x)^3} \, dx$

when you see $\ln x$, think of $\frac{1}{x}$ as its derivative

$u = 2 + \ln x \qquad du = \frac{1}{x}\,dx$

$I = \int (2+\ln x)^{-3} \, \frac{1}{x}\,dx = \int u^{-3} \, du$

$= \dfrac{u^{-2}}{-2} + c$

$= \dfrac{-1}{2(2+\ln x)^2} + c$

5. $I = \int (2x+1)\sqrt{x^2+x+2}\; dx$

$u = x^2+x+2 \quad du = (2x+1)\,dx$

$I = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + c$

$= \frac{2}{3}(x^2+x+2)^{3/2} + c$

6. $I = \int (4x^3-6x)(x^4-3x^2+1)^{18}\; dx$

$u = x^4-3x^2+1 \quad du = (4x^3-6x)\,dx$

$I = \int u^{18}\,du = \frac{(x^4-3x^2+1)^{19}}{19} + c$

IF THE NUMERICAL COEFFICIENT IS OFF
we adjust by a scaling factor

7. $I = \int \sqrt{3x+5}\; dx$

$u = 3x+5 \quad du = \underline{3}\,dx$

The 3 is missing. No worries. In the original integral, there is just a dx, but $dx = \frac{1}{3}du$:

$I = \int u^{\frac{1}{2}}\left(\frac{1}{3}du\right) = \frac{1}{3}\int u^{1/2}\,du$

$= \frac{1}{3}\left(\frac{u^{3/2}}{3/2}\right) + c$

$= \frac{2}{9}u^{3/2} + c$

$= \frac{2}{9}(3x+5)^{3/2} + c$

5.5 D 3

8. $I = \int x(x^2-3)^4\,dx$

$u = x^2-3 \qquad du = 2x\,dx$

in the integral is $x\,dx$, which equals $\frac{1}{2}du$

$I = \int u^4\left(\frac{1}{2}du\right) = \frac{1}{2}\left(\frac{u^5}{5}\right) + c$

$= \frac{1}{10}(x^2-3)^5 + c$

9. $I = \int 5x^3(6x^4+1)^{95}\,dx$

$u = 6x^4+1 \qquad du = 24x^3\,dx$

$x^3\,dx = \frac{1}{24}du$

$I = \int 5u^{95}\left(\frac{1}{24}du\right) = \frac{5}{24}\left(\frac{u^{96}}{96}\right) + c$

$= \frac{5}{24\cdot 96}(6x^4+1)^{96} + c$

10. $I = \int \frac{x^2(x^3+1)^{-11}}{23}\,dx$

$u = x^3+1 \qquad du = 3x^2\,dx$

$x^2\,dx = \frac{1}{3}du$

$I = \int \frac{u^{-11}}{23}\left(\frac{1}{3}du\right) = \frac{1}{69}\left(\frac{u^{-10}}{-10}\right) + c$

$= -\frac{1}{690}(x^3+1)^{-10} + c$

5.5 D 4

- WRONG NUMBER OF X'S

11. $I = \int x^2 (x^4+3)^6 \, dx$

$u = x^4+3 \qquad du = 4x^3 \, dx$

impossible

- TOO MANY X'S

12. $I = \int x^3 \sqrt{x^2-1} \, dx$

$u = x^2-1 \qquad du = 2x \, dx$

$x \, dx = \frac{1}{2} du$

$$I = \int x^2 \sqrt{x^2-1} \, (x\,dx)$$

$$= \int x^2 u^{1/2} \left(\tfrac{1}{2} du\right)$$

$\hookrightarrow$? $\quad x^2 = u+1$

$$I = \tfrac{1}{2} \int (u+1) u^{1/2} \, du$$

$$= \frac{1}{2} \int u^{3/2} + u^{1/2} \, du$$

$$= \frac{1}{2} \left[\left(\frac{u^{5/2}}{5/2} \right) + \left(\frac{u^{3/2}}{3/2} \right) \right] + C$$

$$= \frac{1}{5} u^{5/2} + \frac{1}{3} u^{3/2} + C$$

$$= \frac{1}{5} (x^2-1)^{5/2} + \frac{1}{3} (x^2-1)^{3/2} + C$$

5.5 D 5

13 $I = \int \frac{5x^5}{(x^3+5)^5}\,dx$

$u = x^3 \qquad du = 3x^2\,dx$

$x^2\,dx = \frac{1}{3}\,du$

$I = \int \frac{5x^3\,x^2\,dx}{(x^3+5)^5} = \int \frac{5(x^3)(\frac{1}{3}\,du)}{u^5}$

$x^3 = u - 5$

$$\begin{aligned}
I &= \frac{5}{3}\int (u-5)\,u^{-5}\,du \\
&= \frac{5}{3}\int u^{-4} - 5u^{-5}\,du \\
&= \frac{5}{3}\left(\frac{u^{-3}}{-3} - 5\left(\frac{u^{-4}}{-4}\right)\right) + c \\
&= -\frac{5}{9}(x^3+5)^{-3} + \frac{25}{12}(x^3+5)^{-4} + c
\end{aligned}$$