Section 5.5 part B: Similar Structures

To help understand how the method of substitution works, let's look at some examples where u stays the same and $f'(u)$ changes and some where $f'(u)$ stays the same and u changes. Think of the composite function $f'(u)$ as the 'shell' and u as the 'guts'. That's right - evaluating an integral using substitution is like eating a lobster.

Examples with $u = x^3 + 1$ and different shells

Note that in each case, there needs to be a $3x^2$ so that $du = 3x^2dx$.

$$\begin{aligned}
I &= \int 3x^2 \cos(x^3+1)\,dx = \int \cos(u)\,du \\
I &= \int 3x^2 (x^3+1)^9\,dx = \int u^9\,du \\
I &= \int 3x^2 \sqrt{x^3+1}\,dx = \int \sqrt{u}\,du \\
I &= \int 3x^2 e^{x^3+1}\,dx = \int e^u\,du \\
I &= \int \frac{3x^2}{x^3+1}\,dx = \int \frac{1}{u}\,du
\end{aligned}$$

Examples with $f'(u) = \sin u$ and different values for u

Note that in each case, the u' is different, but we always need to see $u'dx$ to form du.

$$\begin{aligned}
I &= \int 2x \sin(x^2)\,dx = \int \sin(u)\,du \\
I &= \int 3x^2 \sin(x^3)\,dx = \int \sin(u)\,du \\
I &= \int e^x \sin(e^x)\,dx = \int \sin(u)\,du \\
I &= \int \frac{\sin(\ln x)}{x}\,dx = \int \frac{1}{x}\sin(\ln x)\,dx = \int \sin(u)\,du \\
I &= \int \sec^2 x \sin(\tan x)\,dx = \int \sin(u)\,du
\end{aligned}$$