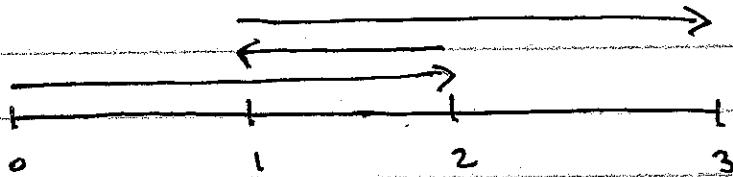


## 5.4 E Distance versus Displacement

Consider an object moving in a straight line back and forth. Travel to the right is positive, and to the left it is negative.

Suppose the object moves as shown below. There are 2 ways to measure its motion



First, the object moves to the right 2 units, then to the left 1 unit, then to the right 2 units.

Distance (how far it has travelled)

$$2 + 1 + 2 = 5 \text{ units} \quad (\text{the motion to the left is included})$$

Displacement (net change in position  
final minus initial)

$$3 - 0 = 3$$

(or  $2 - 1 + 2$ ; the motion to the left detracts from the displacement)

### 5.4 E 2

If the object travels with velocity  $v(t)$ ,  
 it is moving to the right when  $v(t) > 0$   
 and moves to the left when  $v(t) < 0$ .

Since motion to the left counts in the  
 distance, we have

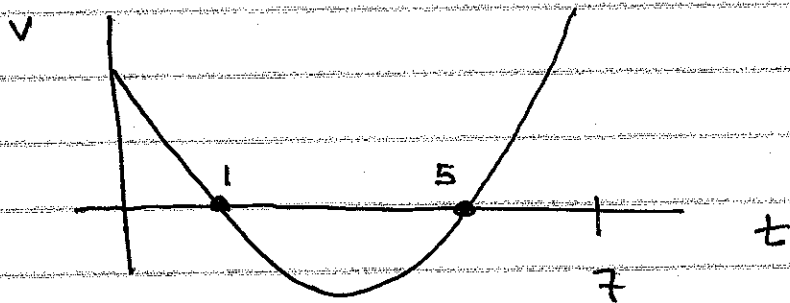
$$\text{distance travelled} = \int_0^a |v(t)| dt$$

since  $|v(t)|$  is always positive

For displacement, the left-moving regions  
 detract, so

$$\text{displacement} = \int_0^a v(t) dt$$

Ex An object moves with velocity  $v(t) = t^2 - 6t + 5$  ft/sec  
 from  $t = 0$  s to  $t = 7$  s. Find the distance  
 it travelled and its displacement



In  $[0, 1]$ ,  $v(t) > 0$  so it's moving to the right  
 $[1, 5]$ ,  $v(t) < 0$  left  
 $[5, 7]$ ,  $v(t) > 0$  right

5.4 E 3

$$\begin{aligned}\text{displacement} &= \int_0^7 t^2 - 6t + 5 \, dt \\ &= \left. \frac{1}{3}t^3 - 3t^2 + 5t \right|_0^7 \\ &= \frac{1}{3} \cdot 7^3 - 3 \cdot 49 + 35 - (0 - 0 + 0) \\ &= 2.33 \text{ ft}\end{aligned}$$

$$\begin{aligned}\text{distance} &= \int_0^7 |t^2 - 6t + 5| \, dt \\ &= \int_0^1 (t^2 - 6t + 5) \, dt + \int_1^5 -(t^2 - 6t + 5) \, dt \\ &\quad + \int_5^7 (t^2 - 6t + 5) \, dt \\ &= \left. \frac{1}{3}t^3 - 3t^2 + 5t \right|_0^1 - \left. \left( \frac{1}{3}t^3 - 3t^2 + 5t \right) \right|_1^5 + \left. \frac{1}{3}t^3 - 3t^2 + 5t \right|_5^7 \\ &= \left( \frac{1}{3} - 3 + 5 \right) - (0 - 0 + 0) \\ &\quad - \left( \frac{1}{3}5^3 - 3 \cdot 25 + 25 \right) + \left( \frac{1}{3} - 3 + 5 \right) \\ &\quad + \left( \frac{1}{3}7^3 - 3 \cdot 49 + 35 \right) - \left( \frac{1}{3}5^3 - 3 \cdot 25 + 25 \right) \\ &= 2.33 - 0 \\ &\quad - (-8.33) + (2.33) \\ &\quad + (2.33) - (-8.33) \\ &= 23.66 \text{ ft}\end{aligned}$$