Section 5.4 part A: the Indefinite Integral

1. The standard notation for writing an antiderivative is called an indefinite integral.

   In Section 4.9, we wrote: if $f'(x) = 2x$ then $f(x) = x^2 + c$

   Starting now, we will write this statement as

   $$\int 2x\,dx = x^2 + c$$

   Notice that there are no limits of integration; that's what makes it 'indefinite'. Remember to always include the $+c$.

2. A few examples include

   $$\begin{aligned} \int x^3 + 4\,dx &= \frac{x^4}{4} + 4x + c \\ \int \frac{3}{1+x^2}\,dx &= 3\tan^{-1} x + c \\ \int \frac{1}{x}\,dx &= \ln|x| + c \end{aligned}$$

3. THE TWO PARTS OF THE FUNDAMENTAL THEOREM OF THE THE CALCULUS

   The 2 parts of the FTC are

   $$\begin{aligned} \frac{d}{dx}\int_a^x f(t)\,dt &= f(x) \\ \int \frac{df}{dx}\,dx &= f(x) + c \end{aligned}$$

   The first part says: start with a function. Integrate it then differentiate it. You end up where you started.

   The second part says: start with a function. Differentiate it then integrate it. You end up where you started.

   The point is the integration and differentiation cancel each other — they are inverses of each other!

4. Warning Number One. Before you find an antiderivative, you have to make sure the integrand is in the proper form. Example:

   $$\int (x-1)(3x+2)\,dx \neq \left(\frac{x^2}{2} - x\right)\left(\frac{3x^2}{2} + 2x\right) + c$$

   Instead, you must simplify the expression before integrating:

   $$\int (x-1)(3x+2)\,dx = \int 3x^2 - x - 2\,dx = x^3 - \frac{x^2}{2} - 2x + c$$

5. Warning Number Two. Functions with absolute values can be tricky. With an indefinite integral, we don't need to worry:

$$\int |x - x^2|\, dx = \left| \frac{x^2}{2} - \frac{x^3}{3} \right| + c$$

but if we look at a definite integral, we have to worry about whether the expression inside the absolute value is positive or negative. Remember, if Y is negative, then $|Y| = -Y$. So consider

$$I = \int_{-1}^{2} |x - x^2|\, dx$$

The function $x - x^2$ has roots at $x = 0, 1$ so the integral must be split into 3 parts:

$$I = \int_{-1}^{2} |x - x^2|\, dx = \int_{-1}^{0} |x - x^2|\, dx + \int_{0}^{1} |x - x^2|\, dx + \int_{1}^{2} |x - x^2|\, dx$$

Now look at $x - x^2$ on each subinterval - it is negative in the first and third subintervals and positive in the middle. This means in the middle integral, $|x - x^2| = -(x - x^2)$, so we get

$$I = \int_{-1}^{2} |x - x^2|\, dx = -\int_{-1}^{0} x - x^2\, dx + \int_{0}^{1} x - x^2\, dx - \int_{1}^{2} x - x^2\, dx$$

The graph of $|x - x^2|$ appears below. We now have 3 integrals to compute. This is painful to contemplate, painful to compute and painful to type, so we're done here.