5.3 F Leibniz' Rule

we can build a composite function using

$$F(x) = \int_a^x f(t)\,dt$$

$$g(x) = F(u(x)) = \int_a^{u(x)} f(t)\,dt$$

$$g(x) = \int_0^{x^2} \sqrt{t^2+1}\,dt$$

$$g(x) = \int_{-2}^{3x+1} \sin(t+1)\,dt$$

To differentiate $g(x)$, use the chain rule.
The result is called Leibniz's Rule

$$\frac{dg}{dx} = \frac{dF}{du} \cdot \frac{du}{dx} = \left[f(u(x)) \right] \left[u'(x) \right]$$

outer inner

Ex 1
$$g(x) = \int_0^{x^2} \sqrt{t^2+1}\,dt$$
$$g'(x) = \left[\sqrt{(x^2)^2+1} \right]\left[2x \right] = 2x\sqrt{x^4+1}$$

Ex 2
$$g(x) = \int_{-2}^{3x+1} \sin(t+1)\,dt$$
$$g'(x) = \left[\sin(\underline{3x+1}+1) \right]\left[3 \right] = 3\sin(3x+2)$$

Ex 3
$$g(x) = \int_{57}^{\sin x} \frac{t}{t^2+2t+7}\,dt$$
$$g'(x) = \left[\frac{\sin x}{\sin^2 x + 2\sin x + 7} \right]\left[\cos x \right]$$

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Ex 4 $g(x) = \int_{x^3}^{5} \ln t + e^t \, dt$

$\int_b^a f(t)\,dt = -\int_a^b f(t)\,dt$

so

$g(x) = -\int_5^{x^3} \ln t + e^t \, dt$

$g'(x) = -\left[\ln x^3 + e^{x^3}\right]\left[3x^2\right]$

general rule : $\frac{d}{dx} \int_{v(x)}^{a} f(t)\,dt$

$= -\left[f(v(x))\right]\left[v'(x)\right]$

Ex 5 $g(x) = \int_{\ln x}^{b} \ln t \, dt$

$g'(x) = -\left[\ln(\ln x)\right]\left[\frac{1}{x}\right]$

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combine :

$g(x) = \int_{v(x)}^{u(x)} f(t)\,dt$

rewrite $g(x) = \int_{v(x)}^{a} f(t)\,dt + \int_a^{u(x)} f(t)\,dt$

so $g'(x) = -f(v(x))v'(x) + f(u(x))u'(x)$

usual form $\frac{d}{dx} \int_{v(x)}^{u(x)} f(t)\,dt$

$= f(u(x))u'(x) - f(v(x))v'(x)$

Ex 6 $g(x) = \int_{x^2}^{x^3} \tan^{-1} t \, dt$

$g'(x) = (\tan^{-1} x^3)\cdot 3x^2 - (\tan^{-1} x^2)\cdot 2x$

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Ex 7 $k(x) = \int_{x^3+x}^{\sin x} \sqrt{t+1}\, dt$

$k'(x) = \sqrt{\sin x+1} \cdot \cos x - \sqrt{x^3+x+1} \cdot (3x^2+1)$