5.3 D Trig and Inverse Trig functions

- Always use radian mode

- recall
$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$, $(-\cos x)' = \sin x$
$(\tan x)' = \sec^2 x$
$(\cot x)' = -\csc^2 x$, $(-\cot x)' = \csc^2 x$
$(\sec x)' = \sec x \tan x$
$(\csc x)' = -\csc x \cot x$, $(-\csc x)' = \csc x \cot x$
$(\tan^{-1} x)' = \frac{1}{1+x^2}$
$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$

Ex 1 $\int_{\pi/2}^{\pi} \cos x \, dx = \sin x \Big|_{\pi/2}^{\pi} = \sin\pi - \sin\frac{\pi}{2} = 0 - 1 = -1$

Ex 2 $\int_{\pi/2}^{\pi} \sin x \, dx = -\cos x \Big|_{\pi/2}^{\pi} = (-\cos\pi) - (-\cos\frac{\pi}{2})$
$= -(-1) - 0 = 1$

Ex 3 $\int_0^{\pi/4} \sec x \tan x \, dx = \sec x \Big|_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0$
$= \frac{1}{\sqrt{2}/2} - \frac{1}{1} = \sqrt{2} - 1$
$\left(\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \right)$

Ex 4 $\int_0^{\pi/4} \sec^2 x \, dx = \tan x \Big|_0^{\pi/4} = \tan\frac{\pi}{4} - \tan 0 = 1 - 0 = 1$

Ex 5 $\int_{\pi/4}^{\pi/2} \csc x \cot x \, dx = -\csc x \Big|_{\pi/4}^{\pi/2} = (-\csc\frac{\pi}{2}) - (-\csc\frac{\pi}{4})$
$= -1 + \sqrt{2}$

5.3 D 2

Ex 6 $\int_{\pi/4}^{\pi/2} \csc^2 x \, dx = -\cot x \Big|_{\pi/4}^{\pi/2} = \left(-\cot\frac{\pi}{2}\right) - \left(-\cot\frac{\pi}{4}\right)$

$= 0 + 1 = 1$

Ex 7 $\int_0^1 \frac{1}{1+x^2}\, dx = \tan^{-1} x \Big|_0^1 = \tan^{-1} 1 - \tan^{-1} 0$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$

Ex 8 $\int_{\frac{\sqrt{2}}{2}}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x \Big|_{\sqrt{2}/2}^{\sqrt{3}/2} = \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{\sqrt{2}}{2}$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12}$

$(\sinh x)' = \cosh x$ $\qquad \sinh 0 = 0$

$(\cosh x)' = \sinh x$ $\qquad \cosh 0 = 1$

Ex 9 $\int_0^5 \sinh x \, dx = \cosh x \Big|_0^5 = \cosh 5 - \cosh 0$

$= \cosh 5 - 1$

Ex 10 $\int_0^7 \cosh x \, dx = \sinh x \Big|_0^7 = \sinh 7 - \sinh 0$

$= \sinh 7$