

5.3 D Trig and Inverse Trig Functions

Always use radian mode

recall

$$\begin{aligned} (\sin x)' &= \cos x \\ (\cos x)' &= -\sin x, \quad (-\cos x)' = \sin x \\ (\tan x)' &= \sec^2 x \\ (\cot x)' &= -\csc^2 x, \quad (-\cot x)' = \csc^2 x \\ (\sec x)' &= \sec x \tan x \\ (\csc x)' &= -\csc x \cot x, \quad (-\csc x)' = \csc x \cot x \\ (\tan^{-1} x)' &= \frac{1}{1+x^2} \\ (\sin^{-1} x)' &= \frac{1}{\sqrt{1-x^2}} \end{aligned}$$

Ex 1 $\int_{\pi/2}^{\pi} \cos x \, dx = \sin x \Big|_{\pi/2}^{\pi} = \sin \pi - \sin \frac{\pi}{2} = 0 - 1 = -1$

Ex 2 $\int_{\pi/2}^{\pi} \sin x \, dx = -\cos x \Big|_{\pi/2}^{\pi} = (-\cos \pi) - (-\cos \frac{\pi}{2})$
 $= -(-1) - 0 = 1$

Ex 3 $\int_0^{\pi/4} \sec x \tan x \, dx = \sec x \Big|_0^{\pi/4} = \sec \frac{\pi}{4} - \sec 0$
 $= \frac{1}{\sqrt{2}/2} - \frac{1}{1} = \sqrt{2} - 1$
 $(\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}})$

Ex 4 $\int_0^{\pi/4} \sec^2 x \, dx = \tan x \Big|_0^{\pi/4} = \tan \frac{\pi}{4} - \tan 0 = 1 - 0 = 1$

Ex 5 $\int_{\pi/4}^{\pi/2} \csc x \cot x \, dx = -\csc x \Big|_{\pi/4}^{\pi/2} = (-\csc \frac{\pi}{2}) - (-\csc \frac{\pi}{4})$
 $= -1 + \sqrt{2}$

5.3 D 2

$$\begin{aligned} \text{Ex 6} \quad \int_{\pi/4}^{\pi/2} \csc^2 x \, dx &= -\cot x \Big|_{\pi/4}^{\pi/2} = \left(-\cot \frac{\pi}{2}\right) - \left(-\cot \frac{\pi}{4}\right) \\ &= 0 + 1 = 1 \end{aligned}$$

$$\begin{aligned} \text{Ex 7} \quad \int_0^1 \frac{1}{1+x^2} \, dx &= \tan^{-1} x \Big|_0^1 = \tan^{-1} 1 - \tan^{-1} 0 \\ &= \frac{\pi}{4} - 0 = \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} \text{Ex 8} \quad \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}} \, dx &= \sin^{-1} x \Big|_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} = \sin^{-1} \frac{\sqrt{3}}{2} - \sin^{-1} \frac{\sqrt{2}}{2} \\ &= \frac{\pi}{3} - \frac{\pi}{4} \\ &= \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12} \end{aligned}$$

$$(\sinh x)' = \cosh x$$

$$\sinh 0 = 0$$

$$(\cosh x)' = \sinh x$$

$$\cosh 0 = 1$$

$$\begin{aligned} \text{Ex 9} \quad \int_0^5 \sinh x \, dx &= \cosh x \Big|_0^5 = \cosh 5 - \cosh 0 \\ &= \cosh 5 - 1 \end{aligned}$$

$$\begin{aligned} \text{Ex 10} \quad \int_0^7 \cosh x \, dx &= \sinh x \Big|_0^7 = \sinh 7 - \sinh 0 \\ &= \sinh 7 \end{aligned}$$