

5.3 B Power functions

$$\begin{aligned}
 \text{Ex 1} \quad I &= \int_2^3 x^3 - 2x + 6 \, dx = \left. \frac{x^4}{4} - x^2 + 6x \right|_2^3 \\
 &= \left(\frac{3^4}{4} - 9 + 18 \right) - \left(\frac{16}{4} - 4 + 12 \right) \\
 &= \frac{81}{4} + 9 - (4 - 4 + 12) \\
 &= \frac{81}{4} - 3 = \frac{81}{4} - \frac{12}{4} = \frac{69}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex 2} \quad I &= \int_0^3 x^3 - 2x + 6 \, dx = \left. \frac{x^4}{4} - x^2 + 6x \right|_0^3 \\
 &= \left(\frac{81}{4} - 9 + 18 \right) - (0 - 0 + 0) = \frac{81}{4} + 9 \\
 &= \frac{117}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex 3} \quad I &= \int_1^{4/3} x^{7/5} \, dx = \left. \frac{x^{12/5}}{12/5} \right|_1^{4/3} \\
 &= \left(\frac{5}{12} \left(\frac{4}{3} \right)^{12/5} - \frac{5}{12} \cdot 1 \right) = \frac{5}{12} \left(\left(\frac{4}{3} \right)^{12/5} - 1 \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex 4} \quad I &= \int_{+3}^8 \frac{1}{3} x^{5/3} - 2x^{1/2} \, dx \\
 &= \left. \frac{1}{3} \frac{x^{8/3}}{8/3} - 2 \frac{x^{3/2}}{3/2} \right|_{+3}^8 \\
 &= \left. \frac{1}{8} x^{8/3} - \frac{4}{3} x^{3/2} \right|_{+3}^8 \\
 &= \left(\frac{1}{8} 8^{8/3} - \frac{4}{3} 8^{3/2} \right) - \left(\frac{1}{8} (3)^{8/3} - \frac{4}{3} \cdot 3^{3/2} \right) \\
 &\quad \hookrightarrow 8^{1/3} = 2 \quad \text{so} \quad 8^{8/3} = 2^8 = 256 \\
 &= \left(\frac{1}{8} \cdot 256 - \frac{4}{3} \cdot 8 \right) - \left(\frac{1}{8} \cdot 3^{8/3} - 4 \cdot 3^{1/2} \right) \\
 &= \left(32 - \frac{32}{3} \right) - \left(\quad \quad \quad \right) \\
 &= \frac{64}{3} - \left(\quad \quad \quad \right)
 \end{aligned}$$

B 2

$$\begin{aligned} \text{Ex 5} \quad I &= \int_2^4 x^{-1/2} + x^{-3/5} dx \\ &= \frac{x^{1/2}}{1/2} + \frac{x^{2/5}}{2/5} \Big|_2^4 \\ &= 2x^{1/2} + \frac{5}{2}x^{2/5} \Big|_2^4 \\ &= \left(2 \cdot 4^{1/2} + \frac{5}{2} \cdot 4^{2/5} \right) - \left(2 \cdot 2^{1/2} + \frac{5}{2} \cdot 2^{2/5} \right) \\ &= 4 + \frac{5}{2} \cdot 4^{2/5} - 2^{3/2} - \frac{5}{2} \cdot 2^{2/5} \end{aligned}$$