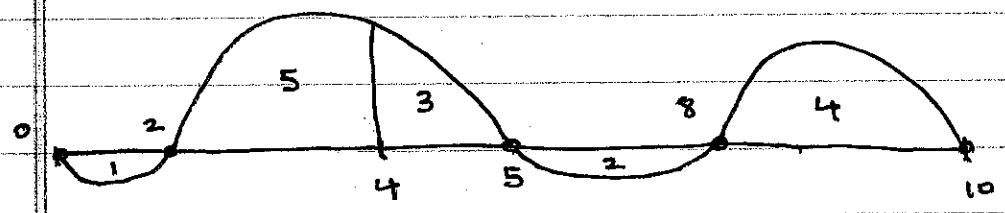
5.2 Properties of Integrals



a $\int_0^2 f(x) = -1$

b $\int_2^5 f(x)\,dx = \int_2^4 f(x)\,dx + \int_4^5 f(x)\,dx = 5 + 3 = 8$

c $\int_8^2 f(x)\,dx = -\int_2^8 f(x)\,dx = -\left[5 + 3 + (-2)\right] = -6$

$(dx < 0)$

d $\int_0^5 f(x)\,dx = \int_0^8 f(x)\,dx - \int_5^8 f(x)\,dx$

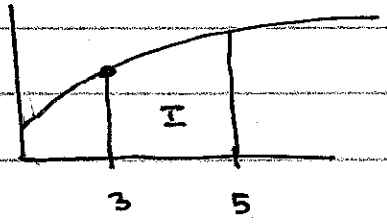
$-1+5+3 = (-1+5+3+(-2)) - (-2)$

$7 = 7$

e $\int_2^5 3f(x)\,dx = 3\int_2^5 f(x)\,dx = 3(5+3) = 24$

f $\int_8^8 f(x)\,dx = 0$

☐ Find a lower bound for $I = \int_3^5 \sqrt{1+x}\, dx$



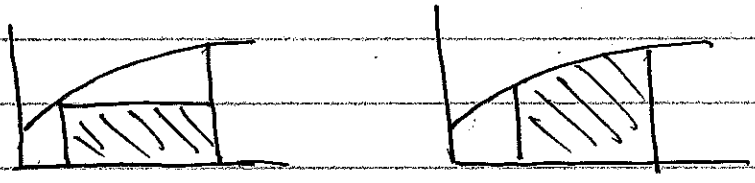
smallest function value : $2 \le \sqrt{1+x}$

(when $x=3$)

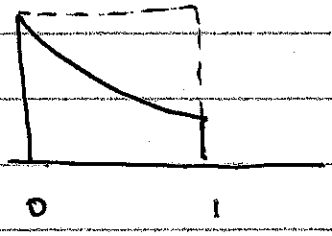
so $\int_3^5 2\, dx \le \int_3^5 \sqrt{1+x}\, dx$

$2(5-3)$

$4 \le \int_3^5 \sqrt{1+x}\, dx$



☐ Find an upper bound for $I = \int_0^1 e^{-x}\, dx$



$e^{-x} \le 1$ for $x \in [0,1]$

so

$\int_0^1 e^{-x}\, dx \le \int_0^1 1\, dx = 1(1-0)$

$= 1$