5.2 Practice Problems

1a $A = \lim_{N\to\infty} \sum_{i=1}^{N} \frac{3}{N}\left(\underbrace{\left(4+\frac{3}{N}i\right)^3} - 2\underbrace{\left(4+\frac{3}{N}i\right)} + 1 \right)$

① $x_i = 4 + \frac{3}{N}i = a + hi = a + \frac{b-a}{N}i$

$a = 4 \quad b-a = 3 \quad$ so $b = 7$

② $f(x_i) = x_i^3 - 2x_i + 1$

③ $A = \int_4^7 x^3 - 2x + 1 \, dx$

1b $A = \lim_{N\to\infty} \sum_{i=1}^{N} \frac{4}{N}\left(\underbrace{\left(-6+\frac{4}{N}i\right)^2} + 6\underbrace{\left(-6+\frac{4}{N}i\right)} \right)$

① $x_i = -6 + \frac{4}{N}i = a + \frac{b-a}{N}i$

$a = -6 \quad b-a = 4 \quad b = -2$

② $f(x_i) = x_i^2 + 6x_i$

③ $A = \int_{-6}^{-2} x^2 + 6x \, dx$

2a $\int_1^4 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^4 f(x)\,dx = 2 + (-1) = 1$

b $\int_1^9 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^4 f(x)\,dx + \int_4^6 f(x)\,dx + \int_6^9 f(x)\,dx$

$= \quad 2 \quad + \quad (-1) \quad + \quad (-6) \quad + \quad 4 \quad = -1$

c $\int_3^6 f(x)\,dx = \int_3^4 f(x)\,dx + \int_4^6 f(x)\,dx = -1 + (-6) = -7$

d $\int_4^9 f(x)\,dx = \int_4^6 f(x)\,dx + \int_6^9 f(x)\,dx = -6 + 4 = -2$

e $\int_6^4 f(x)\,dx = -\int_4^6 f(x)\,dx = -(-6) = 6$

f $\int_9^3 f(x)\,dx = -\int_3^9 f(x)\,dx$

$= -\left[\int_3^4 f(x)\,dx + \int_4^6 f(x)\,dx + \int_6^9 f(x)\,dx \right]$

$= -\left[\quad -1 \quad + \quad (-6) \quad + \quad 4 \right] = -(-3) = 3$

g $\int_4^4 f(x)\,dx = 0$