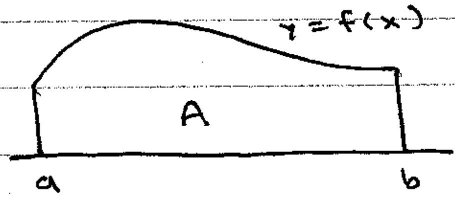
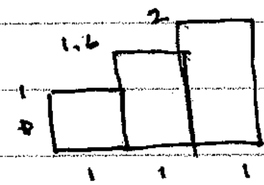
5.1 Area under a curve

a) •



with a curved top, finding A is not trivial

b) • easy area :



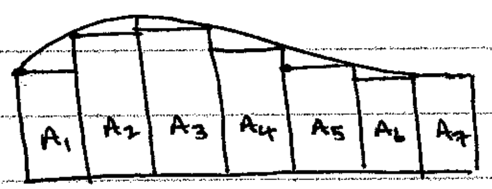
$A = A_1 + A_2 + A_3$

$= 1 \cdot 1 + 1 \cdot 1.6 + 1 \cdot 2$

$= 4.6$

c) Method

Break $[a, b]$ into N subintervals, approximate the area with rectangles



issue: height of each rect.

$$A \approx A_1 + \cdots + A_7 = \sum_{i=1}^{7} A_i$$

d) 2 types of problems

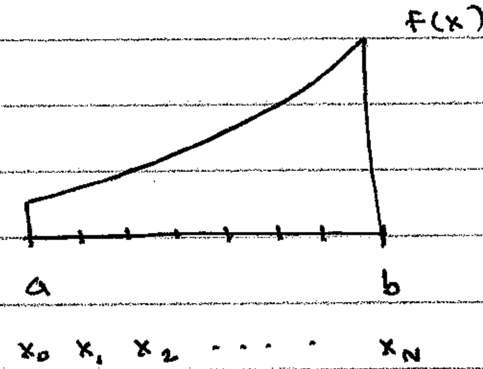
i) Numerical Sums. Approximate A using a finite number (N) of rectangles

ii) Riemann Sums. Get the exact A using infinitely many rectangles $(N \to \infty)$

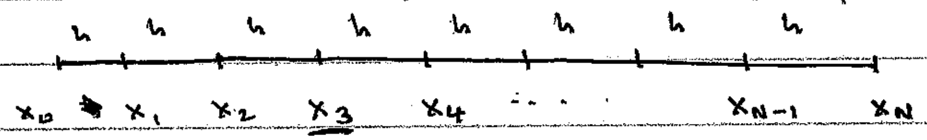
i) $A \approx \sum_{i=1}^{N} A_i$

ii) $A = \lim_{N \to \infty} \sum_{i=1}^{N} A_i$

e) Numerical Sums



The set of x_n values, $\{x_0, x_1, x_2, \cdots, x_n\}$ is a PARTITION. The points are evenly spaced:



$\boxed{h \text{ or } \Delta x}$

$x_3 = x_2 + h$ or $x_3 = a + 3h$

$\Downarrow$ $\Downarrow$

• $x_i = x_{i-1} + h$ • $x_i = a + ih$

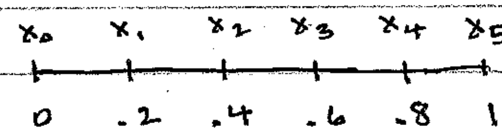
N subintervals means $h = \Delta x = \dfrac{b-a}{N}$

so $x_i = a + \dfrac{(b-a)}{N} i$

f) Ex 1 Partitions

a) $[0,1]$ $N=5$

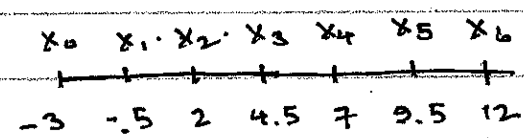
$h = \dfrac{b-a}{N} = \dfrac{1-0}{5} = .2$



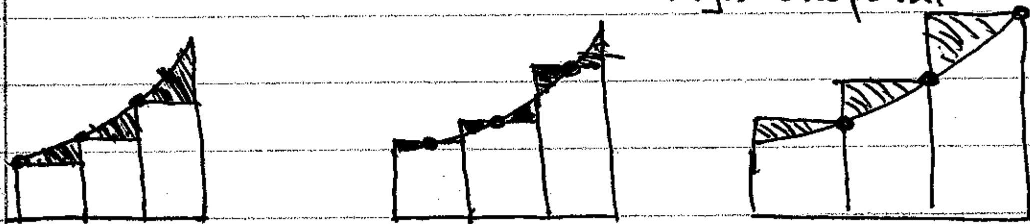
b) $[-3,12]$ $N=6$

$h = \dfrac{b-a}{N} = \dfrac{12-(-3)}{6}$

$= \dfrac{15}{6} = 2.5$



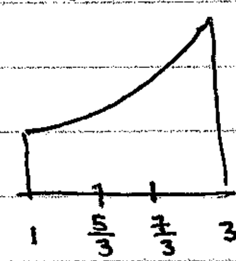
g) For each rectangle, we need a way to pick the height. Typical choices: left endpoint, midpoint, right endpoint



left mid right

shaded regions: error in the approximation

h) Ex 2 $f(x) = x^3 + 4$ on $[1, 3]$ with $N = 3$

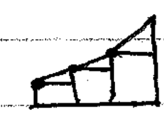


$h = \frac{3-1}{3} = \frac{2}{3}$

- left endpoints

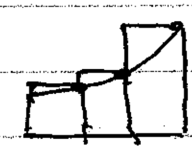
7/3

$$A \approx \frac{2}{3} f(1) + \frac{2}{3} f\left(\frac{5}{3}\right) + \frac{2}{3} f\left(\frac{7}{3}\right)$$
$$= 20.2224$$

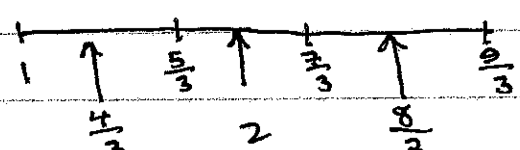


- right endpoints

$$A \approx \frac{2}{3} f\left(\frac{5}{3}\right) + \frac{2}{3} f\left(\frac{7}{3}\right) + \frac{2}{3} f(3)$$
$$= 37.5557$$



- midpoints



$$A \approx \frac{2}{3} f\left(\frac{4}{3}\right) + \frac{2}{3} f(2) + \frac{2}{3} f\left(\frac{8}{3}\right)$$
$$= 27.5556$$

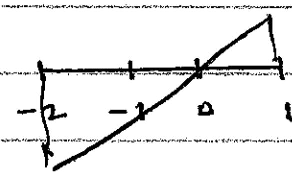
with $N = 50$ left   27.4832

mid   27.9984   actual : 28

right   28.5232

i) Ex 3 $f(x) = x$ on $[-2, 1]$ with $N = 3$

$$h = \frac{1-(-2)}{3} = 1$$

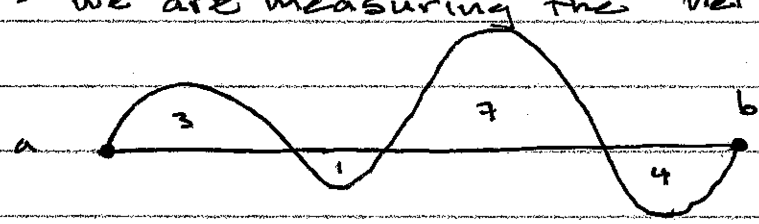


- left $A \approx 1\,f(-2) + 1\,f(-1) + 1\,f(0)$

$$= -2 + (-1) + 0 = -3 \qquad \left(\text{actual: } -\tfrac{3}{2}\right)$$

- ah ha, the area can be negative !

idea : we are measuring the 'net area'



net area $A = +3 - 1 + 7 - 4 = 5$

notation : "definite integral"

$$A = \int_a^b f(x)\, \underline{dx}$$

$\rightarrow$ differential

"the integral from a to b of f(x) dx"

so Ex 2 : $\int_1^3 x^3 + 4 \, dx = 28$

$\int_{-2}^{1} x \, dx = -3/2$